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FOREIGN TECHNOLOGY DIVISION

FLIGHT DYNAMICS

by

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Chapter VII.

CURVILINEAR FLIGHT OF AIRCRAFT.

7.1. Conditions of the emergence of curvilinear motion and its special feature/peculiarity.

During the steady motion of aircraft along straight path, all the acting on it forces are mutually balanced. The disequilibrium centiliter, that act on aircraft, will lead to the appearance of the corresponding accelerations, and aircraft will initiate to accomplish unsteady motion along complex curved path.

In the general case the disequilibrium of external forces leads to the appearance of tangential acceleration (j_t), which coincides with direction of flight, and normal (j_n), perpendicular toward heading (Fig. 7.1). The tangential acceleration, which appears during a change in the thrust/rod, the drags, to weight component in the direction of motion, produces change in the flight speed (retards or accelerates the motion of aircraft). The normal acceleration, which

appears during a change in the hoisting and lateral aerodynamic forces, with the normal weight component of aircraft, leads to a change in the flight-path curvature. Thus, necessary the condition of curvilinear flight is the presence of normal acceleration.

Aerodynamic forces (drag, hoisting and lateral force) they depend on the flight speed, angles of attack, angles and angles of deflection of aerodynamic controllers. Along the conditions of flight safety, the slip angles usually are small and therefore the unbalanced force, which appears during the appearance of a slip, it is obtained also small. The large unbalanced force is obtained, as a rule, as a result of a change in the angle of attack.

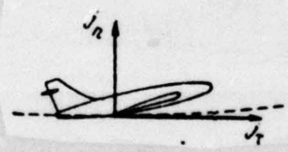

Fig. 7.1. Projections of acceleration on the axis of wind coordinate system in flight of aircraft along curved path.

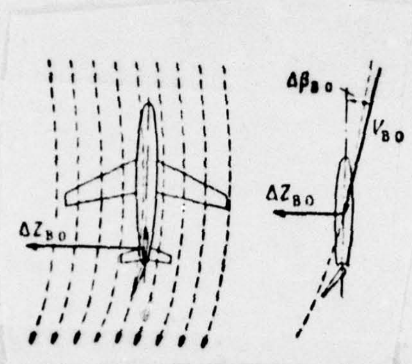

Fig. 7.2. Diagram of the flow about the aircraft during curvilinear flight in horizontal plane.

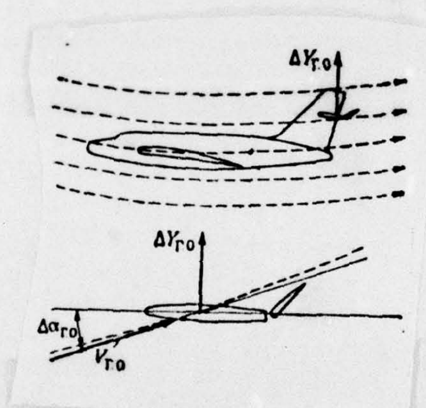

Fig. 7.3. Diagram of the flow about the aircraft during curvilinear flight in vertical plane.

normal acceleration connected with flight speed and the radius of curvature of trajectory by relationship

$$j_n = \frac{V^2}{r}, \qquad (7.1)$$

from which follows that the normal acceleration grow/rises with an increase in the velocity of flight and a decrease in the radius of curvature of trajectory.

In common/general/total the case flat/plane curvilinear motion can be accomplished in any plane. For the aircraft of the civil

aviation the greatest interest they represent curvilinear motions in vertical plane, for example under the conditions of alignment/levelling, and in horizontal - during the fulfillment of bank/turns. Flow lines in the turned motion of the air flow, which flows around aircraft during its motion along curved paths in horizontal and vertical planes, are bent (Figs. 7.2 and 6.3), which leads to a change in the local true angles of attack and slip of the different cell/elements of aircraft. These changes the more greater, the further is arranged the cell/element of aircraft from its center of mass. The greatest changes in the angles of attack and slip will occur of tail assembly. So, in flight of aircraft along curved path in horizontal plane (Fig. 7.2) as a result of the distortion of flow lines slip angle of vertical tail assembly will change by the value of $\Delta\beta_{в.о.}$ which, in turn, will lead to the appearance of the supplementary lateral force of the $\Delta Z_{в.о.}$ which impedes the rotation of aircraft. In flight of aircraft along curved path in vertical plane (Fig. 7.3) appears the supplementary lift of the $\Delta Y_{г.о.}$ also inhibiting to the rotation of aircraft.

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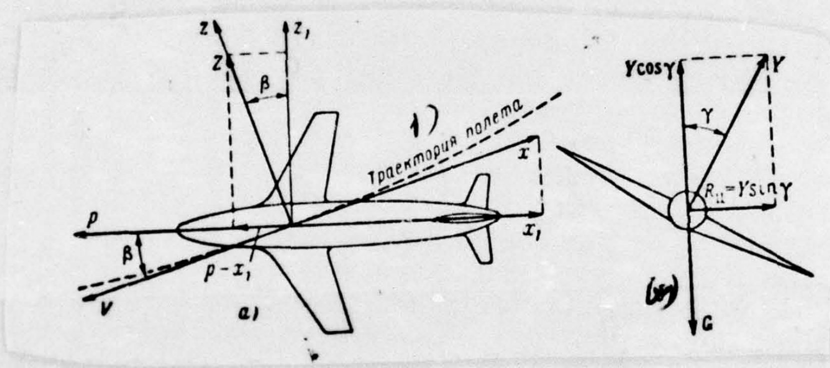

Fig. 7.4. Diagram of the forces, acting to the aircraft: a) during the fulfillment of bank/turn with slip without bank; b) during the fulfillment of bank/turn with bank without slip (standard rate turn).

Key: (1). Flight trajectory.

And in that, and in another case for the preservation/retention/maintaining of curvilinear motion is necessary the supplementary deflection of controls to the side of rotation.

7.2. Curvilinear motion in horizontal plane. Bank/turn and its characteristic.

Curvilinear movement in horizontal plane is called bank/turn. Bank/turn can be fulfilled with slip without bank, with bank without slip, and also with simultaneous slip and bank.

Bank/turn with slip is realized by the deflection of rudder, in this case on control, is created the torque/moment, which turns aircraft. The appearance of a slip leads to the emergence of the lateral force of Z, which determines centripetal force (Fig. 7.4a).

The centripetal force, with bank/turn only with bank, is determined by the projection of lift on horizontal plane (Fig. 7.4b). Since lift in this case will be more than the weight of aircraft, bank/turn with bank can be realized either by an increase in the

angle of attack with the preservation/retention/maintaining of flight speed or by an increase in the velocity of flight at the fixed angle of attack. An increase in the angles of attack is restricted to the possibility of flow separation from wing and the loss by the aircraft of stability (usually these phenomena begin to be exhibited with $\alpha > \alpha_{эк}$); therefore bank/turn, as a rule, it is fulfilled at increasing speed.

With bank/turns with simultaneous slip and bank, the centripetal force is determined by the sum of the projections of the lateral and lift forces on horizontal plane. With slip to external plane, the lateral force increases centripetal force (Fig. 7.5a), with slip by internal (Fig. 7.5b) - it decreases.

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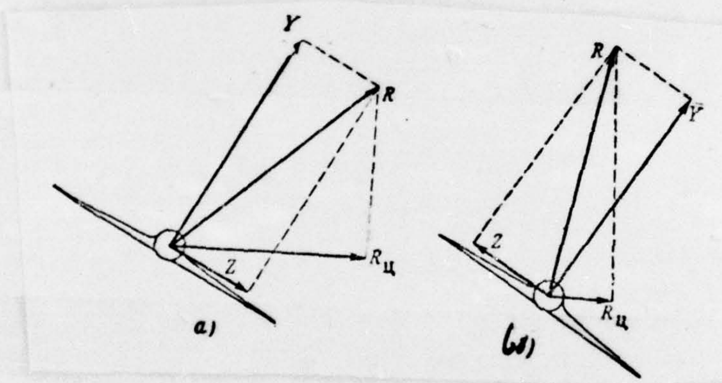

Fig. 7.5. Diagram of the forces, which act on aircraft during the fulfillment of bank/turn with bank with external (a) and internal (b) slips.

The fundamental characteristics of bank/turn are the radius of bank/turn and the time of the fulfillment of bank/turn, necessary for the fulfillment of turn on 2π is glad. The values of radius and time of bank/turn are determined by the value of centripetal force.

Centripetal force is connected with speed and radius of bank/turn by formula

$$R_{ц}=\frac{mV^2_{вир}}{r_{вир}} \quad (7.2)$$

and consequently:

$$r_{вир}=\frac{mV^2_{вир}}{R_{ц}}. \quad (7.3)$$

If bank/turn it is fulfilled at constant velocity, then the time of bank/turn

$$t_{\text{вир}} = \frac{2\pi r_{\text{вир}}}{V_{\text{вир}}}. \qquad (7.4)$$

7.3. Standard rate turn.

Of all forms of bank/turns simplest is the calculation of the so-called correct bank/turn, i.e., bank/turn only with bank and without slip. As it follows, from Fig. 7.4b, with the correct steady turn they occur of equality

$$R_{ц}=Y\sin\gamma,\quad G=Y\cos\gamma$$

or taking into account relationship (7.2)

$$\left.\begin{aligned}\frac{mV^2_{вир}}{r_{вир}}&=Y\sin\gamma,\\ G&=Y\cos\gamma.\end{aligned}\right\}\qquad(7.5)$$

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According to (2.14) the ratio of lift to the gross weight of aircraft equal to the normal load factor of n_y; therefore from the second equality of system (7.5) it is possible to obtain relationship

$$n_y = \frac{1}{\cos\gamma}, \qquad (7.6)$$

from which it follows that for the preservation/retention/maintaining of motion in horizontal plane with an increase in the normal load factor must increase the roll attitude. Furthermore, from equality (7.6) it is evident also that the bank/turn with attitude $\gamma = \pi/2$ is glad in horizontal plane cannot be carried out, since it requires $n_y = \infty$.

System (7.5) makes it possible to determine all the

characteristics of the standard rate turn through the g-force. So, after dividing piecemeal the first equality into the second and taking into account that $\mathrm{tg}\gamma=\sqrt{n_y^2-1}$, we will obtain

$$r_{\text{вир}}=\frac{V^2_{\text{вир}}}{g\sqrt{n_y^2-1}}. \qquad (7.7)$$

after substituting this value of $r_{\text{вир}}$ in (7.4), we will obtain expression for determining the time of the bank/turn:

$$t_{\text{вир}}=\frac{2\pi}{g}\,\frac{V_{\text{вир}}}{\sqrt{n_y^2-1}}. \qquad (7.8)$$

velocity on bank/turn also depends on g-force and can be

determined from the first equation of system (7.5), if we into it substitute the value of $r_{\text{вир}}$ according to (7.7) and the value of lift

$$Y=c_y \frac{\varrho V^2_{\text{вир}}}{2} S.$$

producing the indicated substitution, we will obtain the following expression for a velocity on the bank/turn:

$$V_{\text{вир}}=\sqrt{2\frac{G}{S}\frac{n_y}{\varrho c_y}}. \qquad (7.9)$$

then since in level straight flight

$$V_{\text{гор}}=\sqrt{\frac{2G}{\varrho S c_y}},$$

$$\frac{V_{\text{вир}}}{V_{\text{гор}}}=\sqrt{n_y}, \qquad (7.10)$$

i.e. velocity on bank/turn into the $\sqrt{n_y}$ of times the more appropriate speed level straight flight.

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An increase in the velocity of flight leads to an increase in the required power on bank/turn. For a steady turn

$$P_{п.вир} = X_{вир},$$
$$G = Y_{вир} \cos \gamma,$$

hence

$$P_{п.вир} = G \frac{X_{вир}}{Y_{вир} \cos \gamma}$$

or taking into account (7.6)

$$P_{\text{п.вир}}=\frac{X_{\text{вир}}}{Y_{\text{вир}}}Gn_y=\frac{Gn_y}{K_{\text{вир}}}. \tag{7.11}$$

with one and the same angles of attack and small mach numbers lift-drag ratio in rectilinear level flight and on bank/turn will be identical; therefore

$$\frac{P_{\text{п.вир}}}{P_{\text{п.гор}}}=n_y, \tag{7.12}$$

i.e. the required thrust in standard rate turn of the n_y of times more required thrust during level flight.

Since the required power of $N_{\text{п}}=P_{\text{п}}V_{\text{п}}$,

$$\frac{N_{\text{п.вир}}}{N_{\text{п.гор}}}=\frac{P_{\text{п.вир}}V_{\text{вир}}}{P_{\text{п.гор}}V_{\text{гор}}}=n_y^{3/2}. \qquad (7.13)$$

thus, required power on bank/turn into the $n_y^{3/2}$ of times is more required power for a straight-and-level flight.

7.4. Maximum bank/turn.

As it follows from expressions (7.7) and (7.8), to decrease radius and time of standard rate turn possible, after increasing g-force. But the increase in the g-force will lead to an increase in the required power.

in this case, the required power can prove to be more than had, and the fulfillment of bank/turn in horizontal plane will turn out to be impossible.

Therefore in order to judge the possibility of the fulfillment of steady turn with one g-force or the other (roll attitude), on the grid of required powers, executed for a level flight, is constructed the grid of the required and available powers on bank/turn.

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According to expressions (7.10) and (7.13) required power on bank/turn can be expressed thus:

$$N_{п.вир}=\frac{N_{п.гор}}{V^3_{гор}}V^3_{вир}\ . \qquad (7.14)$$

$$\frac{N_{\text{п.гор}}}{V_{\text{гор}}^3} = A(\alpha),$$

To each angle of attack corresponds the completely determined value of relation

$$N_{\text{п.вир}} = AV_{\text{вир}}^3 . \qquad (7.15)$$

which can be found from curved required power for a level a straight flight (Fig. 7.6). Therefore expression (7.14) can be rewritten in the form

To the grid of powers, by utilizing expressions (7.15) and (7.10), it is possible to apply the cubic parabola ab, which characterizes a change in the required power for the fulfillment of bank/turn with the assigned angle of attack. It is obvious, to each angle of attack it will correspond the determined parabola, and to each point of parabola from a to b will correspond completely determined the g-force and roll attitude.

At point b, the required for a bank/turn power will be equal to the maximally possible available power for the given height/altitude. The g-force and the roll attitude, that correspond to this point, are determined from expression

$$n_{\text{пр}}=\frac{V^2_{\text{нпр.пр}}}{V^2_{\text{гор}}}; \tag{7.16}$$

$$\gamma_{\text{пр}}=\arccos\frac{1}{n_{\text{пр}}} \tag{7.17}$$

and are extreme values of n and γ, with which it is possible to carry out a standard rate turn with the assigned angle of attack at given height/altitude.

The bank/turn, fulfilled from $\gamma_{\text{пр}}$, is called maximum. A radius and the time of maximum bank/turn can be found from relationships (7.7) and (7.8), if we substitute in them the $n=n_{\text{пр}}$:

$$r_{\text{пр}}=\frac{V^2_{\text{гор}}}{g}\,\frac{n_{\text{пр}}}{\sqrt{n^2_{\text{пр}}-1}}; \tag{7.18}$$

$$t_{\text{пр}}=\frac{2\pi V_{\text{гор}}}{g}\sqrt{\frac{n_{\text{пр}}}{n^2_{\text{пр}}-1}}. \tag{7.19}$$

Fig. 7.6. To the determination of the velocity of standard rate turn with the aid of the curved available and required powers for a level flight.

Fig. 7.7. Curves of required powers for a standard rate turn.

Key: (1). kW. (2). ~~in grad~~ rad. (3). km/h.

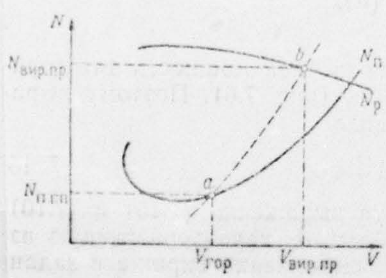

Fig. 7.8.

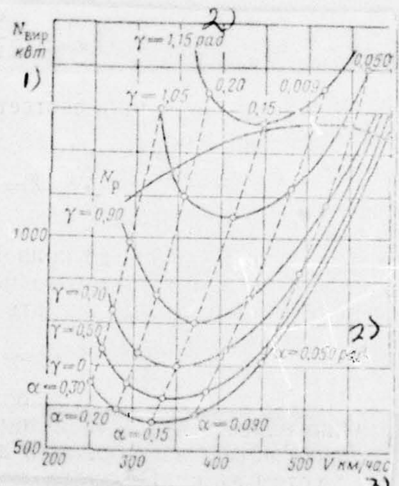

Fig. 7.7.

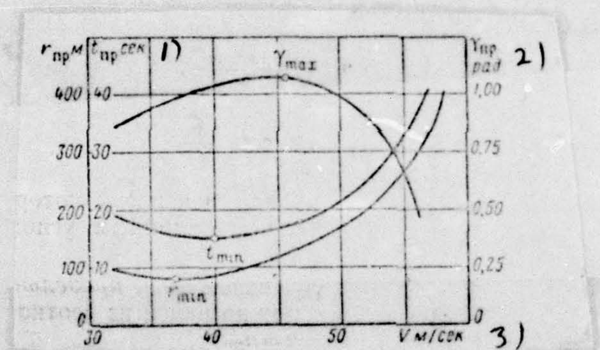

Fig. 7.8. Characteristics of maximum bank/turn.

Key: (1). s. (2). rad [redacted] (3). m/s.

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For the analysis of the possible modes of bank/turn in all speed range and angles of attack of is plotted the grid of parabolas (Fig. 7.7). After selecting on each parabola of the points, which correspond to the determined roll attitude, it is possible to construct the curves of required powers for the different roll attitudes, which make it possible to examine all the possible mode/conditions of bank/turn at the given height/altitude. So, for example, Fig. 7.7 shows that the standard rate turn with bank $\gamma = 1.15$ rad to carry out at the given height/altitude impossibly.

By using curve/graph (Fig. 7.7) and formulas (7.17), (7.18), (7.19), it is possible to calculate the extreme values of γ_{np}, r_{np}, t_{np} and to construct the diagram of their dependence from velocity on bank/turn (Fig. 7.8).

As it follows from diagram, the maximum value of γ_{max} and the minimum values of r_{min} and t_{min} correspond to the completely definite velocities, whereupon the velocities, which correspond to these extreme points, are different. This is explained by the fact that the

$\gamma_{\text{пр}}$ depends only on g-force, whereas $t_{\text{пр}}$ and $r_{\text{пр}}$ they depend even on flight speed.

in practice the mode/conditions, which correspond $r_{\min}$ and $t_{\min}$, are not realized, since corresponding it angles of attack prove to be more than $\alpha_{\text{эк}}$, but with $\alpha=\alpha_{\text{эк}}$ are possible flow separations from wing and loss by the aircraft of stability of motion. therefore the most advantageous mode/conditions of bank/turn corresponds $\gamma_{\max}$.

The bank of aircraft on bank/turn, besides limitation according to power, can be limited and on others parameters. so, for example, for passenger aircraft bank on bank/turn frequently is restricted to the value of the permissible normal load factor.

7.5. curvilinear motion in vertical plane.

The curvilinear motions of the aircraft of the civil aviation in vertical plane occur upon transition from level flight toward reduction/descent, and also in the stage of alignment/levelling during landing on runway. The motions of aircraft in these flight

conditions are being unsteady.

As an example let us examine the flight of aircraft in the stage of alignment/levelling. The curvilinear unsteady flight in vertical plane is described by equations (2.14). Taking into account that with alignment/levelling the flight is made with reduction/descent, let us change in the equations of system (2.10) sign and θ to reverse/inverse, i.e., during reduction/descent let us angle θ consider positive, and then equations accept the following form:

$$\left.\begin{aligned} P+G\sin\theta-X&=m\frac{dV}{dt},\\ Y-G\cos\theta&=-mV\frac{d\theta}{dt}.\end{aligned}\right\} \qquad (7.20)$$

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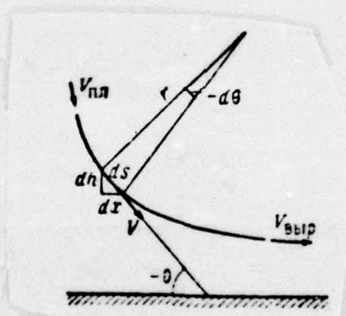

Fig. 7.9. Kinematic cell/elements of the curvilinear flight of aircraft in vertical plane.

In order to investigate the motion of aircraft, it is necessary to integrate system (7.20). Taking into account that the thrust/rod of aircraft on landing usually strongly is throttled, so that it it is possible to consider equal to zero, and after dividing both parts of equation (7.20) piecemeal by weight of aircraft, we will obtain the following system of equations:

$$\left.\begin{aligned} \sin\theta-\frac{n_y}{K}&=\frac{1}{g}\frac{dV}{dt};\\ n_y-\cos\theta&=-\frac{V}{g}\frac{d\theta}{dt}.\end{aligned}\right\} \quad (7.21)$$

the cell/element of arc dS, over which moves the aircraft taking into account angle θ during reduction/descent (Fig. 7.9) can be written as

$$ds=-r\,d\theta, \quad (7.22)$$

$$n_y - \cos\theta = \frac{1}{g}\,\frac{V^2}{r}. \qquad (7.25)$$

taking into account which the second equation of system (7.21) it is possible to rewrite in the form

$$r = \frac{V^2}{g\,(n_y - \cos\theta)}. \qquad (7.26)$$

equation (7.25) makes it possible to determine the radius of curvature of the arc:

during the performance calculation of alignment/levelling radius of curvature usually is taken as as constant, to the half-sum of its values in the beginning $(V=V_{пл},\ \theta=\theta_{пл})$ and at the end $(V=V_{выр},\ \theta\approx 0)$ of the alignment/levelling:

$$r_{cp}=\frac{1}{2g}\left(\frac{V_{пл}^2}{n_y-\cos\theta_{пл}}+\frac{V_{выр}^2}{n_y-1}\right). \quad (7.27)$$

this simplification makes it possible to integrate system (7.21)

in the final form and to obtain the comparatively simple approximation formulas for the characteristics of alignment/levelling.

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Let us find the speed of flight at the end of the alignment/levelling. For this purpose, we convert the first equation of system (7.21) taking into account (7.24) to form

$$\sin\theta-\frac{n_y}{K}=-\frac{V}{gr}\,\frac{dV}{d\theta}. \tag{7.28}$$

after separation of variables we will obtain equation

$$-dV^2=2gr\left(\sin\theta-\frac{n_y}{K}\right)d\theta,$$

integrating which in apparitors from $\theta=\theta_{пл}$ to $\theta_{выр}$ and from $V=V_{пл}$ to $V_{выр}$ taking into account the constancy of radius r, let us have

$$V_{пл}^2-V_{выр}^2=2gr_{ср}\left[\frac{n_y}{K}(\theta_{пл}-\theta_{выр})-(\cos\theta_{выр}-\cos\theta_{пл})\right]. \quad (7.29)$$

by set/assuming in equation (7.29) $\theta_{выр}=0,$ we will obtain expresssion for a speed at the end of the alignment/levelling:

$$V^2_{\text{выр}} = V^2_{\text{пл}} - 2gr_{\text{ср}}\left[\frac{n_y}{K}\theta_{\text{пл}} - (1-\cos\theta_{\text{пл}})\right]. \qquad (7.30)$$

solving (7.30) together with (7.27), let us have

$$\left.\begin{aligned} \frac{V^2_{\text{выр}}}{V^2_{\text{пл}}} &= \frac{(n_y-1)\left[1+\left(1-\frac{\theta_{\text{пл}}}{K}\right)n_y - 2\cos\theta_{\text{пл}}\right]}{(n_y-\cos\theta_{\text{пл}})\left[\cos\theta_{\text{пл}}+\left(1+\frac{\theta_{\text{пл}}}{K}\right)n_y - 2\right]}, \\ r_{\text{ср}} &= \frac{V^2_{\text{пл}}}{2g}\,\frac{2n_y - 2\cos\theta_{\text{пл}} - 1}{(n_y-\cos\theta_{\text{пл}})\left[\cos\theta_{\text{пл}}+\left(1+\frac{\theta_{\text{пл}}}{K}\right)n_y - 2\right]}. \end{aligned}\right\} \qquad (7.31)$$

let us find now the time, necessary for a change in the flight path angle from $\theta=\theta_{пл}$ to θ = 0. From expression (7.24) we have

$$t=-\int_{\theta_{пл}}^{0}\frac{r}{V}\,d\theta. \tag{7.32}$$

if we consider that the alignment/levelling is conducted at certain average speed

$$V_{ср}=\frac{1}{2}\,(V_{пл}+V_{выр}),$$

that from expression (7.32) it is possible to obtain simple formula for determining time t:

$$t \approx \frac{r_{cp}}{V_{cp}} \theta_{\text{на}}. \qquad (7.33)$$

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For the more precision determination of recovery time integral (7.32) must be calculated taking into account expression (7.29).

A change in altitude during alignment/levelling taking into account angle θ (Fig. 7.9) can be defined as

$$dh = -\sin\theta ds,$$

whence after the substitution of value ds according to formula (7.22)

$$dh = r\sin\theta d\theta. \qquad (7.34)$$

integrating expression (7.34) in apparitors from $\theta = \theta_{пл}$ to $\theta_{выр} = 0$ and from $h = H_{пл}$ to $h = H_{выр}$, we obtain

$$\Delta H = H_{пл} - H_{выр} = r_{ср}(1 - \cos\theta_{пл}). \qquad (7.35)$$

according to formulas (7.31), (7.33), (7.35) it is possible to approximately calculate the characteristics of alignment/levelling.

the analysis of formulas (7.31), (7.35) shows that with an increase of velocity and gliding angle grow/rises the height/altitude of the beginning of alignment/levelling. It it is possible to decrease by means of an increase in g-force and decrease in the lift-drag ratio. Since the value of the permissible g-force usually is limited, decreases in the height/altitude of the beginning of alignment/levelling frequently attain for a decrease in the lift-drag ratio, making alignment/levelling with the released interceptor/spoilers, by the air brakes and other attachments, which increase the drag of aircraft. PROBLEMS FOR REPETITION.

1. Which conditions are necessary for the flight path curvature of the flight of aircraft? As they are realized in practice?

2. how does affect slip angle a radius of bank/turn?

3. Why required for the fulfillment of bank/turn thrust/rod and power are greater than during level flight?

4. Why it is not possible to make steady turn with roll attitude, large maximum?

PROBLEMS.

1. Aircraft flies horizontally with the speed of $V_{гор}=500$ km/h. Which it is necessary to create the g-force of n_y, in order to fulfill standard rate turn with a radius of $r_{вир}=4000$ m? Which will be required time for the fulfillment with this of overload complete bank/turn?

Answer/response: $n_y=1,15$; t = 2 min 16 s.

2. Aircraft accomplishes at height/altitude H = 1000 m standard rate turn with a radius of $r_{вир}=5000$ m. To determine, how one should increase its bank in order to decrease a radius of bank/turn down to the value of $r_{вир}=4200$ m? Lift coefficient, necessary for fulfillment at this height/altitude of the steady level flight, to accept equal $c_{y\,гор}=0,3$, load on wing G/S = 3920 N/m².

The answer/responds: to 0.105 rad.

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Chapter VIII

GENERAL FORMULATION OF THE PROBLEM OF STABILITY AND AIRCRAFT HANDLING.

8.1. The basic concepts and determinations.

During the analysis of the motion of the center of mass, were examined the special cases when rotation relative to the center of mass is absent and the main vector of the external forces, which act on aircraft, always is applied in its center of mass, i.e., the main moment of external forces is equal to zero.

In the general case, which corresponds to the actual conditions of flight, the external forces, which act on aircraft, lead to the emergence of torque/moments, and therefore for determining the behavior of aircraft under actual conditions necessary research on the joint motion of the center of mass and rotation/revolution around it under the action of forces and torque/moments.

In this case, it is necessary to keep in mind that the forces and the torque/moments, which act on aircraft, depend on the velocity of the motion of the center of mass and rotation/revolution around it, since during the rotation/revolution of the around of the center of mass change the angles of attack and slip and, consequently, also the forces, which determine the trajectory of the motion of the

center of mass. In their turn, the value of total aerodynamic force and the center-of-pressure location depend on trajectory speed. Therefore with the change of flight speed, appears the supplementary torque/moment, which causes a change in the kinematic parameters of the rotation/revolution of aircraft around the center of mass.

The motion of the center of mass along the predetermined trajectory is called the basic motion, and the forces, which realize this motion, they are called by the basic forces.

Under actual conditions to aircraft, at certain points in time, besides the basic forces, considered during the trajectory calculation of motion, can act the secondary forces, which produce change in the parameters of the basic (not disturbed) motion. These perturbing forces are or random or subordinate to the determined law as a function of time. The phenomena, which cause the emergence of these forces, are called the perturbation factors.

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By examples of the perturbation factors are atmospheric turbulence, fuel jettisoning, a change in the position of the center of mass, brought about the displacement/movement of the passengers over aircraft in flight, the engine failure, the random control displacement etc.

With the perturbation factors the perturbing forces appear directly and indirectly. In the first case the emergence of perturbing forces and torque/moments is not the consequence of a change in the kinematic parameters of motion. By examples can serve the random failure of engine, the pulsation of the thrust/rod of power plant, the probable deviations organs of control.

In the second case the perturbing forces and torque/moments appear as a result of a change in the flight speed, angles of attack and slip and other kinematic parameters of the motion of aircraft, caused by the physical phenomena. for example in the presence of vertical currents in the turbulent atmosphere periodically change the angles of attack of wing and horizontal tail assembly, as a result of which appear the perturbation aerodynamic forces and torque/moments.

the changed under the action of the perturbation impetus unlike basic is called the disturbed motion.

Let the basic motion be determined by the velocity of the motion of the center of mass, by the angles of attack α, of bank γ, of pitch ϑ, of slip β, of yawing by and other kinematic parameters, which are the determined functions of time. Under the action of perturbing forces, these parameters of the motion of aircraft will undergo change. The difference in the disturbed motion from basic (not disturbed) is determined by changes (variations) in the velocity ΔV, in the angles of attack $\Delta\alpha$, of the bank $\Delta\gamma$, of the slip $\Delta\beta$, of the pitch $\Delta\vartheta$, of the yawing Δ and of other kinematic parameters of motion. Variations in the kinematic parameters of motion $\Delta\vartheta$, $\Delta\alpha$, $\Delta\gamma$, ... are called another disturbance/perturbations (for example a variation in the velocity is called disturbance velocity).

The parameters of the disturbed movement at any point in time can be defined as sum of the corresponding parameter of the undisturbed motion and its disturbance/perturbation: $\vec{V}=\vec{V}_0+\Delta\vec{V}$, $\vec{\alpha}=\vec{\alpha}_0+\Delta\vec{\alpha}$, $\vec{\beta}=\vec{\beta}_0+\Delta\vec{\beta}$, $\vec{\gamma}=\vec{\gamma}_0+\Delta\vec{\gamma}$, $\vec{\vartheta}=\vec{\vartheta}_0+\Delta\vec{\vartheta}$, $\vec{\psi}=\vec{\psi}_0+\Delta\vec{\psi}$, $\vec{\theta}=\vec{\theta}_0+\Delta\vec{\theta}$.

In the general case of the value of disturbance/perturbations $\Delta\vec{\vartheta}$, $\Delta\vec{\alpha}$, $\Delta\vec{V}$, $\Delta\vec{\beta}$, $\Delta\vec{\gamma}$, $\Delta\vec{\psi}, \ldots$ they can be different. In given course are examined predominantly the slight disturbances, which much the less corresponding parameters of the basic motion (for example $\vec{V}_0 \gg \Delta\vec{V}$).

The basic problem of the stability theory of motion is research on the disturbed motion.

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If after the break-down of the perturbation factors of a variation in the parameters of motion $\Delta\vartheta$, $\Delta\alpha$, $\Delta\beta$, ... in the course of time, decreasing, they become the less preassigned value, then the state of motion or equilibrium is stable. But if the variations in the parameters of motion $\Delta\vartheta$, $\Delta\alpha$, $\Delta\beta$, ... in the course of time grow/rise unlimitedly or they approach the final limit, which exceeds the assigned magnitude, then the initial state of motion or equilibrium is unstable.

Let us examine the stability of equilibrium in an example of pendulum with the upper suspension (Fig. 8.1a) or of the ball/sphere in cup (Fig. 8.1b). At the lower point of cup, the angle ϕ between vertical line and common/general/total normal to the surface of ball/sphere and cup is equal to zero and, therefore, the weight of ball/sphere completely is balanced by the reaction of surface.

If by the action of perturbing force ball/sphere is moved to the right, then angle ϕ differs from its zero value for the value of disturbance/perturbation $\Delta\phi$. After the break-down of perturbing force, force component of weight G sine ϕ approaches to decrease the angle ϕ down to zero and thereby to return ball/sphere the starting position of equilibrium. Consequently, ball/sphere at the lower point of cup has the stable form of equilibrium. Analogously it is possible to demonstrate the stability of the equilibrium of pendulum with the upper suspension. The ability of body or system to approach starting position or state of motion after the break-down of disturbance/perturbations is called static stability. The effect of the resisting forces and friction contributes to damping oscillation/vibrations and to the restoration/reduction of the initial state of equilibrium.

If disturbance/perturbations are great, then ball/sphere can of one stable position of equilibrium pass to another (position C in Fig. 8.1b). This means that the equilibrium of ball/sphere at point A is stable with respect to low and unstably with respect to the finite disturbances.

By examples of the unstable form of equilibrium they can serve the ball/sphere, which lies on convex surface (Fig. 8.2a), or pendulum in the upper vertical position (Fig. 8.2b). As can be seen from figures, under the action of the low perturbing forces not only is disturbed equilibrium, but, furthermore, the tangential component of weight after the break-down of perturbing force attempts to take away body (ball/sphere) further from the previous position of equilibrium, i.e., to increase angle ϕ.

With an increase in the radius of curvature of concave surface (Fig. 8.1b) the value of restoring force G sine ϕ decreases and, as a result, descends the rate of the return of ball/sphere to the starting position of equilibrium. By increasing radius of curvature to infinity, we will obtain the horizontal plane which will be the boundary between the stable and unstable forms of the equilibrium of ball/sphere. This form of equilibrium is called indifferent.

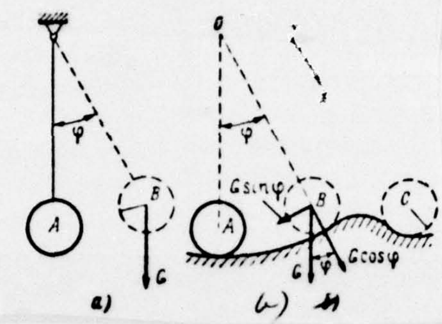

Fig. 8.1. The stable equilibrium of pendulum with the upper suspension (a) and ball/sphere in indentation on rigid surface (b).

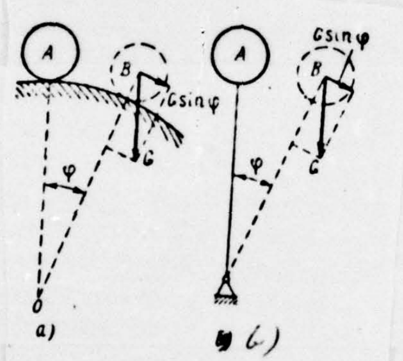

Fig. 8.2. Unstable equilibrium of pendulum with lower suspension (b) and ball/sphere on solid convex surface (a).

The static stability is the necessary, but not sufficient condition common/general/total or the so-called transient stability of motion. It characterizes only initial tendency toward the restoration/reduction of the initial motion or equilibrium. For the possibility of more complete analysis, it is necessary to trace the disturbance/perturbations. If disturbance/perturbations in the course of time vanish, or become the less preassigned low value, motion stable.

If ball/sphere, being located in point B (Fig. 8.1b), it approaches position of equilibrium, then in the absence of the force of friction it will not defined for long oscillate about point A, without being stopped in it. Under these conditions of static stability, proves to be insufficiently in order to return ball/sphere to state of rest at point A.

The behavior of the basic forces during the action of the perturbation factors on aircraft lays out of its equilibrium and motion. Let in the initial undisturbed motion of force and the torque/moments, which act on aircraft, be completely balanced. In this case they say that the aircraft is balanced. under the action of the perturbation factors, can change the angle of attack, flight

speed and another kinematic parameters of the motion of aircraft. If changes in the kinematic parameters of motion are contribute to the emergence of righting moments and forces, which attempt to remove disturbance/perturbations, then aircraft will be statically is stable. Otherwise the aircraft will be statically is unstable.

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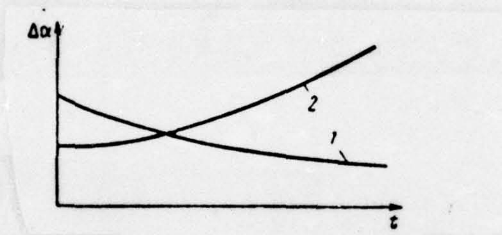

Fig. 8.3. Variations in the angle of attack for stable (1) and unstable (2) aircraft.

The manifestation of transient stability is the tendency to zero of variations in the parameters of the motion of aircraft in the course of time after the break-down of the perturbation factors. The stability level is determined by the speed of a decrease in the variations in the parameters in time. The faster attenuate the variations in the parameters of motion in time, the stabler the aircraft. The degree of the instability of aircraft is determined by the speed of the increase of variations in the parameters of motion in time after the break-down of the perturbation factors. For determining the stability level and degree of instability, they plot a curve of the dependences of a variation in the parameters of motion from time (Fig. 8.3).

Aircraft handling is tightly connected with a change in the forces and torque/moments, which act on aircraft. Conditionally differentiate two form of the controllability: static and dynamic. the static control determines the actions, necessary either for the balance of aircraft in the assigned mode/conditions or for transition from one mode/conditions to another not allowing for the disturbed motion, caused by application to the aircraft of control forces and torque/moments. The dynamic controllability determines the same actions taking into account the transfer disturbed motion, caused by control forces and torque/moments.

Control forces and torque/moments are created by the aircraft controls.

8.2. the equation of the disturbed motion.

In body coordinate system the motion of the aircraft it is described by equations (1.22) - (1.24), to which are added three coupling equations, which escape/ensue of the kinematic relationship

$$
\left.
\begin{aligned}
&m\left(\frac{dV_x}{dt}+\omega_y V_z-\omega_z V_y\right)=R_x,\\
&m\left(\frac{dV_y}{dt}+\omega_z V_x-\omega_x V_z\right)=R_y,\\
&m\left(\frac{dV_z}{dt}+\omega_x V_y-\omega_y V_x\right)=R_z,\\
&J_x\frac{d\omega_x}{dt}+(J_z-J_y)\omega_y\omega_z+J_{xy}\left(\omega_x\omega_z-\frac{d\omega_y}{dt}\right)=M_x,\\
&J_y\frac{d\omega_y}{dt}+(J_x-J_z)\omega_z\omega_x-J_{xy}\left(\omega_y\omega_z+\frac{d\omega_x}{dt}\right)=M_y,
\end{aligned}
\right\}\quad (8.2)
$$

$$
\left.
\begin{aligned}
&J_z\frac{d\omega_z}{dt}+(J_y-J_x)\omega_x\omega_y+J_{xy}(\omega_y^2-\omega_x^2)=M_z,\\
&\omega_x=\frac{d\gamma}{dt}+\frac{d\psi}{dt}\sin\vartheta,\\
&\omega_y=\frac{d\psi}{dt}\cos\vartheta\cos\gamma+\frac{d\vartheta}{dt}\sin\gamma,\\
&\omega_z=\frac{d\vartheta}{dt}\cos\gamma-\frac{d\psi}{dt}\cos\vartheta\sin\gamma,\\
&V_x=V\cos\beta\cos\alpha,\\
&V_y=-V\cos\beta\sin\alpha,\\
&V_z=V\sin\beta.
\end{aligned}
\right\}\quad (8.1)
$$

in the given system of equations instead of the sums of the projections of forces and torque/moments are used the projections of the main vectors of the acting forces and torque/moments, furthermore, index "1" is lowered, since subsequently is utilized the only body coordinate system. Enter the right sides of first six equations of the projection of the main vector of $\vec{R}$ (the force of periphery of R_x, the normal force of R_y, the transverse force of R_z) and of the projection of the main torque/moment of $\vec{M}$ ** omission are the functions of time and kinematic parameters of motion. System of equations (8.1) describes as the initial, undisturbed motion which can be that which was being unsteady, so also airplane disturbance.

*omission -- (rolling moment M_x, yawing moment M_y and pitching moment M_z)

The analysis of stability of motion entails the search of the solution of system (8.1) for not disturbed and agitated motions and the subsequent study of these solutions. If in the course of time the solution to the equations of the disturbed motion has as its asymptote the solution to the equations of the initial motion, then this testifies to the stability of the basic motion (frequently this property is called asymptotic stability).

System of equations (8.1) fairly complicated; therefore for the analysis of the disturbed motion, it is simplified by linearization.

The essence of linearization entails the fact that the disturbance/perturbations are set/assumed low. in this case the disturbed motion differs little from the initial undisturbed motion. All the kinematic and dynamic parameters of the disturbed motion can be represented in the form of the sum of the parameters of the initial motion and corresponding disturbance/perturbations:

$$\left.\begin{aligned} \vartheta&=\vartheta_0(t)+\Delta\vartheta(t), & \alpha&=\alpha_0(t)+\Delta\alpha(t),\\ \gamma&=\gamma_0(t)+\Delta\gamma(t), & \psi&=\psi_0(t)+\Delta\psi(t),\\ V_x&=V_{x0}(t)+\Delta V_x(t), & \omega_x&=\omega_{x0}(t)+\Delta\omega_x(t), \end{aligned}\right\} \quad (8.2)$$

$$\left.\begin{aligned} V_y&=V_{y0}(t)+\Delta V_y(t), & \omega_y&=\omega_{y0}(t)+\Delta\omega_y(t),\\ V_z&=V_{z0}(t)+\Delta V_z(t), & \omega_z&=\omega_{z0}(t)+\Delta\omega_z(t),\\ R_x&=R_{x0}(t)+\Delta R_x(t), & M_x&=M_{x0}(t)+\Delta M_x(t),\\ R_y&=R_{y0}(t)+\Delta R_y(t), & M_y&=M_{y0}(t)+\Delta M_y(t),\\ R_z&=R_{z0}(t)+\Delta R_z(t), & M_z&=M_{z0}(t)+\Delta M_z(t). \end{aligned}\right\} \quad (8.2)$$

First terms in the right parts of the equalities determine the parameters of the undisturbed motion, the second, i.e., the disturbance/perturbations, which characterize the deviations of flight from the initial mode/conditions.

Since in flight range the angles of attack and slip are small (α < 0.2 rad; $|\beta|$ < 0.2 rad) the latter three equations of system (8.1) are simplified:

$$V_x \approx V, \ V_y \approx -V\alpha, \ V_z = V\beta. \tag{8.3}$$

during the linearization of equations (8.1) let us assume to be variations in all kinematic parameters of motion, and also the angles of attack and slip low and to disregard all terms, which contain their products, and by the degrees of the second and above as by low higher order quantities in comparison with other terms of equations.

This assumption together with relationship (8.3) allows the latter three equations (8.1) to write in that which was linearized form:

$$\left.\begin{aligned} V_x &= V_0+\Delta V, \\ V_y &= -V_0\alpha_0-V_0\Delta\alpha, \\ V_z &= V_0\beta+V_0\Delta\beta. \end{aligned}\right\} \quad (8.4)$$

For further simplification in the system of equations (8.1) we assume that the initial undisturbed motion is the rectilinear stable flight and to effect on the aircraft of perturbing forces rotation/revolution around the center of mass is absent, i.e., V_0, α_0, β_0, γ_0, ϑ_0, ϕ_0 it are absent constant values, and $\omega_{x0}=\omega_{y0}=\omega_{z0}=0$. Then the variations in the angular velocities are equal to the quite angular velocities and are determined by relationships

$$\left.\begin{aligned} \Delta\omega_x &= \omega_x = \frac{d\Delta\gamma}{dt}+\frac{d\Delta\psi}{dt}\sin\phi_0, \\ \Delta\omega_y &= \omega_y = \frac{d\Delta\psi}{dt}\cos\vartheta_0\cos\gamma_0+\frac{d\Delta\vartheta}{dt}\sin\gamma_0, \\ \Delta\omega_z &= \omega_z = \frac{d\Delta\vartheta}{dt}\cos\gamma_0-\frac{d\psi}{dt}\cos\vartheta_0\sin\gamma_0. \end{aligned}\right\} \quad (8.5)$$

on the basis this same assumption derivatives of V_x, V_y, V_z in terms of time are equal to:

$$\left.\begin{aligned}\frac{dV_x}{dt}&=\frac{d\Delta V}{dt},\\ \frac{dV_y}{dt}&=-V_0\frac{d\Delta\alpha}{dt},\\ \frac{dV_z}{dt}&=+V_0\frac{d\Delta\beta}{dt}.\end{aligned}\right\}\quad(8.4')$$

Substituting expressions (8.4), (8.4') and (8.5) in equations (8.1), reject/throwing the small second-order quantities and taking into account that the parameters of the basic motion must satisfy the initial equations, we will obtain the following system of equations of the disturbed motion:

$$\left.\begin{aligned}m\frac{d\Delta V}{dt}+mV\beta\omega_y+mV\alpha\omega_z&=\Delta R_x,\\ -m\alpha\frac{d\Delta V}{dt}-mV\frac{d\Delta\alpha}{dt}+mV\omega_z-mV\beta\omega_x&=\Delta R_y,\\ m\beta\frac{d\Delta V}{dt}+mV\frac{d\Delta\beta}{dt}-mV\alpha\omega_x-mV\omega_y&=\Delta R_z,\\ J_x\frac{d\omega_x}{dt}-J_{xy}\frac{d\omega_y}{dt}&=\Delta M_x,\\ J_y\frac{d\omega_y}{dt}-J_{xy}\frac{d\omega_x}{dt}&=\Delta M_y,\\ J_z\frac{d\omega_z}{dt}&=\Delta M_z,\\ \omega_x&=\frac{d\Delta\gamma}{dt}+\frac{d\Delta\psi}{dt}\sin\vartheta,\\ \omega_y&=\frac{d\Delta\psi}{dt}\cos\vartheta\cos\gamma+\frac{d\Delta\vartheta}{dt}\sin\gamma,\\ \omega_z&=\frac{d\Delta\vartheta}{dt}\cos\gamma-\frac{d\Delta\psi}{dt}\cos\vartheta\sin\gamma.\end{aligned}\right\}\quad(8.6)$$

for the simplicity of recording in equations (8.6) is lowered index "by 0" at the values, belonging to the initial motion, and mark "Δ" for angular velocities, since latter are equal to the appropriate increases (in the initial mode/conditions rotation/revolution is absent).

In order that linearization is bygone finished, right part one six equations necessary to present in the form of the explicit functions of variations in the kinematic parameters of the disturbed motion and their derivatives, and also the functions of the coordinates, which characterize surface position, aerodynamic brakes, the engine power rating etc.

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Being based on the analysis of the effect of different factors on the values of the main vector and main torque/moment, to the projection of the main vector and main torque/moment on the axis of body coordinate system it is possible in the first approximation, to present in the form of functions

$$
\left.\begin{aligned}
R_x &= R_x(\alpha,\ \vartheta,\ \beta,\ \varrho,\ V,\ \delta_{\text{в}},\ \delta_{\text{н}},\ \delta_{\text{э}},\ \delta_{\text{р}}),\\
R_y &= R_y(\alpha,\ \vartheta,\ \varrho,\ \beta,\ V,\ \delta_{\text{в}},\ \delta_{\text{р}}),\\
R_z &= R_z(\alpha,\ \vartheta,\ \varrho,\ \beta,\ \gamma,\ \delta_{\text{н}}),\\
M_x &= M_x(\alpha,\ \vartheta,\ \varrho,\ \beta,\ \omega_x,\ \omega_y,\ \delta_{\text{э}},\ \delta_{\text{н}}),\\
M_y &= M_y(\alpha,\ \vartheta,\ \varrho,\ \beta,\ \omega_x,\ \omega_y,\ \delta_{\text{н}},\ \delta_{\text{э}}),\\
M_z &= M_z(\alpha,\ \gamma,\ \vartheta,\ \varrho,\ \beta,\ \omega_z,\ \delta_{\text{в}},\ \delta_{\text{р}}),
\end{aligned}\right| \qquad (8.7)
$$

where the $\delta_{\text{в}}, \delta_{\text{н}}, \delta_{\text{э}}, \delta_{\text{р}}$ - respectively the elevator angles, rudder, ailerons, control lever of the engine power rating.

In relationships (8.7) are taken into account the basic factors,

which affect dynamic motion characteristics. In special cases there can be the changes. For example when using air brakes must be taken into account their effect on the force intensity and torque/moments; in flight of multiengine aircraft with unsymmetric thrust/rod, the value of M_y will depend on the engine power rating, which is characterized by the parameter of δ_p, supplementary resistance etc. Increases in the dynamic motion characteristics can be found, by expanding functions (8.7) in Taylor series and by leaving in expansion/decomposition members, who contain variations to the first degree, for example:

$$\Delta R_x=\frac{\partial R_x}{\partial \alpha}\Delta\alpha+\frac{\partial R_x}{\partial V}\Delta V+\frac{\partial R_x}{\partial \varrho}\Delta\varrho+\frac{\partial R_x}{\partial \vartheta}\Delta\vartheta+$$
$$+\frac{\partial R_x}{\partial \delta_{н}}\Delta\delta_{н}+\frac{\partial R_x}{\partial \delta_{э}}\Delta\delta_{э}+\frac{\partial R_x}{\partial \delta_{p}}\Delta\delta_{p}+\Delta R_{x\,в}.$$

the value of $\Delta R_{xв}$ corresponds to the disturbance/perturbations of the longitudinal force, not caused by the

disturbance/perturbations of the kinematic parameters of motion. It can be either the random function of a change in the atmospheric parameters, turbulence and, etc or the known function of time in fuel dumping, booster ignition, the uncoupling of glider/airframe etc.

Similar expansions in Taylor series can be fulfilled, also, for other dynamic characteristics, and then to introduce the obtained results into the right sides of the corresponding equations of system (8.6).

Let us accept one additional simplifying assumption - about the constancy of air density (ρ = const), since during a small change in altitude it is possible to disregard density effect on the character of the disturbed motion.

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During the recording of the linearized equations, let us use the simplified symbolism for the partial derivatives in terms of rule

$$\frac{\partial \zeta}{\partial \eta} = \zeta \eta.$$

Furthermore, is solved three last/latter equations of system (8.6) relative to derivatives of angles γ, ϑ and ψ.

The made observations and assumptions make it possible to arrive at the following form of the recording of the linearized system of equations of the disturbed motion:

$$
\left.
\begin{aligned}
& m\frac{d\Delta V}{dt}+mV\beta\omega_y+mV\alpha\omega_z-R_x^V\Delta V-R_x^\alpha\Delta\alpha-\\
& -R_x^\vartheta\vartheta-R_x^\beta\Delta\beta=R_x^{\delta_\text{в}}\Delta\delta_\text{в}+R_x^{\delta_\text{н}}\Delta\delta_\text{н}+R_x^{\delta_\text{э}}\Delta\delta_\text{э}+R_x^{\delta_\text{р}}\Delta\delta_\text{р}+\Delta R_{x\,\text{в}},\\
& -m\alpha\frac{d\Delta V}{dt}-mV\frac{d\Delta\alpha}{dt}+mV\omega_z-mV\beta\omega_x-R_y^V\Delta V-\\
& -R_y^\alpha\Delta\alpha-R_y^\vartheta\Delta\vartheta-R_y^\beta\Delta\beta=R_y^{\delta_\text{в}}\Delta\delta_\text{в}+R_y^{\delta_\text{р}}\Delta\delta_\text{р}+\Delta R_{y\,\text{в}},\\
& m\beta\frac{d\Delta V}{dt}+mV\frac{d\Delta\beta}{dt}-mV\alpha\omega_x-mV\omega_y-R_z^V\Delta V-R_z^\alpha\Delta\alpha-\\
& -R_z^\beta\Delta\beta-R_z^\gamma\Delta\gamma=R_z^{\delta_\text{н}}\Delta\delta_\text{н}+\Delta R_{z\,\text{в}},\\
& J_y\frac{d\omega_x}{dt}-J_{xy}\frac{d\omega_y}{dt}-M_x^V\Delta V-M_x^\alpha\Delta\alpha-M_x^\beta\Delta\beta-\\
& -M_x^{\omega_x}\omega_x-M_x^{\omega_y}\omega_y=M_x^{\delta_\text{э}}\Delta\delta_\text{э}+M_x^{\delta_\text{н}}\Delta\delta_\text{н}+\Delta M_{x\,\text{в}},\\
& J_y\frac{d\omega_y}{dt}-J_{xy}\frac{d\omega_x}{dt}-M_y^V\Delta V-M_y^\alpha\Delta\alpha-M_y^\beta\Delta\beta-\\
& -M_y^{\omega_x}\omega_x-M_y^{\omega_y}\omega_y=M_y^{\delta_\text{н}}\Delta\delta_\text{н}+M_y^{\delta_\text{э}}\Delta\delta_\text{э}+\Delta M_{y\,\text{в}},\\
& J_z\frac{d\omega_z}{dt}-M_z^V\Delta V-M_z^\alpha\Delta\alpha-M_z^{\dot\alpha}\frac{d\Delta\alpha}{dt}-M_z^\beta\Delta\beta-\\
& -M_z^{\omega_z}\omega_z=M_z^{\delta_\text{в}}\Delta\delta_\text{в}+M_z^{\delta_\text{р}}\Delta\delta_\text{р}+\Delta M_{z\,\text{в}},\\
& \frac{d\Delta\gamma}{dt}=\omega_x-\omega_y\operatorname{tg}\vartheta\cos\gamma+\omega_z\operatorname{tg}\vartheta\sin\gamma,\\
& \frac{d\Delta\vartheta}{dt}=\omega_y\sin\gamma+\omega_z\cos\gamma,\\
& \frac{d\Delta\psi}{dt}=\frac{\omega_y\cos\gamma-\omega_z\sin\gamma}{\cos\vartheta}.
\end{aligned}
\right\}
\quad (8.8)
$$

system of equations (8.8) is the linearized system of equations (8.1), linear relative to variations in the kinematic parameters.

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If the undisturbed motion is the steady rectilinear flight, then the coefficients with kinematic variations are constant values. When the undisturbed motion - unsteady flight, the coefficients of the linearized equations they are the functions of time, determined from the equations of the undisturbed motion. in general form the exact solution of such systems is difficult at present. In engineering practice are applied the different approximation methods of solution. These questions exceed the scope of manual and can be studied on special literature[1].

FOOTNOTE [1]. V. V. Solodovnikov. Technical cybernetics, book 2. M., "machine-building", 1967. ENDFOOTNOTE.

The solution of system (8.8) makes it possible to trace variations in the kinematic parameters in time dependence, and also

on governing factors and random disturbances. If we assign to aircraft the initial disturbances of the kinematic parameters and to examine further flight under the action only of basic forces and torque/moments (governing factors and random disturbances are absent), then for the stable aircraft of a variation in the kinematic parameters they must in the course of time vanish or remain less than any, preassigned low value.

However, the solution of this system is extremely cumbersome and connected with definite difficulties; therefore tales are developed the methods of the study of stability of motion for the general case of nonlinear differential equations, the which do not require searches their solutions. Majority of these methods is based on the common/general/total stability theory of motion, created by outstanding Russian mathematician and the mechanic A. M. Lyapunovs whose presentation exceeds the scope of the present course.

For examined here cases of slight disturbances, the study of stability of motion is conducted by the simpler methods which are set forth below.

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8.3. Analysis of the equations of the disturbed motion. Longitudinal and yawing motions.

Let us examine in body coordinate system airplane disturbance, which has the plane of symmetry xOy, called longitudinal. In this plane act the force of periphery of R_x the normal force of R_y and the pitching moment of M_z. The displacement/movement of the center

of mass and the rotation/revolution of aircraft around it into the planes of symmetry, taken together, are called axial motion and are characterized by the flight speed of V, by the angles of attack α and of pitch ϑ. In the transverse plane yOz, acts the moment of roll of M_x; the motion in this plane is characterized by the angular velocity of ω_x and by attitude γ. The lateral force Z and the yawing moment M_y act in plane xOz, but the motion in it is characterized by the velocity of V_z, by the angular velocity of ω_y and by the yaw angle ψ.

According to expressions (8.3) and (8.4) the component of vector of the velocity of V_z and its variation in the ΔV_z are determined respectively by angle β and by its variation and trajectory speed.

Planes xOz and yOz, the perpendicular to fore-and-aft plane symmetries xOy, are called the lateral planes; force Z and torque/moments of M_x and M_y acting in these planes, respectively they are united by the name of the lateral group. The rolls and yawing, which proceed in these planes and which are characterized by the angular velocities of ω_x, ω_y, by angles β, γ, ψ and by the trajectory speed of V, in their totality are called yawing motion.

In a strict setting the longitudinal and yawing motion is mutually connected. However, in the cases of slight disturbances axial motion can be examined separately from the lateral. The proof of the validity of this assumption given in I. V. Ostoslavskiy's book "aerodynamics of aircraft" [1] exceeds the scope of this manual.

FOOTNOTE [1]. I. V. Ostoslavskiy. Aerodynamics of aircraft. M., Oborongiz, 1957. ENDFOOTNOTE.

The essence of this assumption entails the fact that during slight disturbances and one and the same flight speed the lateral force and torque/moments do not depend on the kinematic parameters of axial motion, and the longitudinal forces and torque/moments are not caused by the kinematic parameters of yawing motion.

The independence of the longitudinal forces and torque/moments from a change in the kinematic parameters of yawing motion is determined by the following relationships:

$$R_x^{\beta}=R_x^{\omega_x}=R_x^{\omega_y}=R_y^{\beta}=R_y^{\omega_x}=R_y^{\omega_y}=M_z^{\beta}=M_z^{\omega_x}=M_z^{\omega_y}=0.$$

In a similar manner is determined the assumption of the independence lateral torque/moment also strengths from the kinematic parameters of the axial motion:

$$Z^{\alpha}=Z^{\vartheta}=M_x^{\alpha}=M_y^{\alpha}=M_x^{\vartheta}=M_y^{\vartheta}=0.$$

Under these assumptions in the equations of system (8.8), that describe motion in fore-and-aft plane, will disappear the terms, which contain variations in the kinematic parameters of yawing motion, and in the equations, which describe yawing motion, they will disappear the terms, which contain variations in the parameters of axial motion. If we still consider that for contemporary transport aircraft the product of inertia of J_{xy} is very short in comparison with the axial moments of inertia of J_x, J_y, J_z that the system of

differential equations (8.8) it is possible to divide into two independent systems of equations.

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The system of equations of the longitudinal disturbed motion:

$$\left.\begin{aligned}
& m\frac{d\Delta V}{dt} - R_x^V \Delta V - R_x^\alpha \Delta\alpha - R_x^\vartheta \Delta\vartheta = \\
& = R_x^{\delta_\text{в}} \Delta\delta_\text{в} + R_x^{\delta_\text{н}} \Delta\delta_\text{н} + R_x^{\delta_\text{э}} \Delta\delta_\text{э} + R_x^{\delta_\text{р}} \Delta\delta_\text{р} + \Delta R_{x\,\text{в}}, \\
& mV\frac{d\Delta\theta}{dt} - mV\frac{d\Delta\alpha}{dt} - R_y^V \Delta V - R_y^\alpha \Delta\alpha - R_y^\vartheta \Delta\vartheta = \\
& = R_y^{\delta_\text{в}} \Delta\delta_\text{в} + \Delta R_{y\,\text{в}}, \\
& J_z\frac{d^2\Delta\vartheta}{dt^2} - M_z^V \Delta V - M_z^\alpha \Delta\alpha - M_z^{\dot{\alpha}}\frac{d\Delta\alpha}{dt} - M_z^{\omega_z}\frac{d\Delta\vartheta}{dt} = \\
& = M_z^{\delta_\text{в}} \Delta\delta_\text{в} + M_z^{\delta_\text{р}} \Delta\delta_\text{р} + \Delta M_{z\,\text{в}}, \\
& \vartheta = \alpha + \theta.
\end{aligned}\right\} \quad (8.9)$$

The last/latter equation expresses the kinematic constraint between the pitch angles, attack and path inclination.

System of equations of the lateral disturbed motion:

$$\left.\begin{aligned}
&mV\frac{d\Delta\beta}{dt}-mV\alpha\omega_x-mV\omega_y-R_z^{\beta}\Delta\beta-R_z^{\gamma}\Delta\gamma=\\
&\qquad=R_z^{\delta_{\text{н}}}\Delta\delta_{\text{н}}+\Delta R_{z\,\text{в}},\\
&J_x\frac{d\omega_x}{dt}-M_x^{\beta}\Delta\beta-M_x^{\omega_x}\omega_x-M_x^{\omega_y}\omega_y=\\
&\qquad=M_x^{\delta_{\text{э}}}\Delta\delta_{\text{э}}+M_x^{\delta_{\text{н}}}\Delta\delta_{\text{н}}+\Delta M_{x\,\text{в}},\\
&J_y\frac{d\omega_y}{dt}-M_y^{\beta}\Delta\beta-M_y^{\omega_x}\omega_x-M_y^{\omega_y}\omega_y=\\
&\qquad=M_y^{\delta_{\text{н}}}\Delta\delta_{\text{н}}+M_y^{\delta_{\text{э}}}\Delta\delta_{\text{э}}+M_{y\,\text{в}},\\
&\frac{d\Delta\gamma}{dt}=\omega_x-\omega_y\,\mathrm{tg}\,\vartheta\cos\gamma,\\
&\frac{d\Delta\psi}{dt}=\frac{\omega_y\cos\gamma-\omega_z\sin\gamma}{\cos\vartheta}
\end{aligned}\right| \quad (8.10)$$

In the equations of the longitudinal disturbed motion, is excluded the alternating/variable ω_z, since of $\omega_z \approx \frac{d\Delta\vartheta}{dt}$, furthermore, are reject/thrown in view of smallness the terms, which contain the products of the angle of attack or slip angle during a variation in the kinematic parameters.

Systems of equations (8.9) and (8.10) make it possible to examine the disturbed motion, in process of which on aircraft act governing as well as external disturbance/perturbations, when the law of a change in them in time is known.

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In this course in essence, are analyzed the properties of the so-called proper motion of aircraft, i.e., the disturbed motion, caused by the disturbance/perturbation only of initial kinematic parameters. In this case, it is assumed that governing disturbance/perturbations no (control "pressed") and external disturbance/perturbations are absent. In this case the basic forces and the torque/moments, which act on aircraft, undergo change only due to the presence of variations in the kinematic parameters, caused

them by the initial disturbance. For this case of system (8.9) and (8.10) it is possible to write in the following form.

System of equations of the longitudinal disturbed motion:

$$\left.\begin{aligned} &m\frac{d\Delta V}{dt}-R_x^V\Delta V-R_x^\alpha\Delta\alpha-R_x^\vartheta\Delta\vartheta=0,\\ &mV\left(\frac{d\Delta\vartheta}{dt}-\frac{d\Delta\alpha}{dt}\right)-R_y^V\Delta V-R_y^\alpha\Delta\alpha-R_y^\vartheta\Delta\vartheta=0,\\ &J_z\frac{d^2\Delta\vartheta}{dt^2}-M_z^{\omega_z}\frac{d\Delta\vartheta}{dt}-M_z^\alpha\frac{d\Delta\alpha}{dt}-M_z^V\Delta V-M_z^\alpha\Delta\alpha=0,\\ &\vartheta=\theta+\alpha. \end{aligned}\right\} \quad (8.11)$$

System of equations of the lateral disturbed motion:

$$\left.\begin{aligned}
&mV\frac{d\Delta\beta}{dt}-mV\alpha\omega_x-mV\omega_y-k_z^{\beta}\Delta\beta-R_z^{\gamma}\Delta\gamma=0,\\
&J_x\frac{d\omega_x}{dt}-M_x^{\beta}\Delta\beta-M_x^{\omega_x}\omega_x-M_x^{\omega_y}\omega_y=0,\\
&J_y\frac{d\omega_y}{dt}-M_y^{\beta}\Delta\beta-M_y^{\omega_x}\omega_x-M^{\omega_y}\omega_y=0,\\
&\frac{d\Delta\gamma}{dt}=\omega_x-\omega_y\,\mathrm{tg}\,\vartheta\cos\gamma,\\
&\frac{d\Delta\psi}{dt}=\frac{\omega_y\cos\gamma-\omega_z\sin\gamma}{\cos\vartheta}.
\end{aligned}\right\}\qquad(8.12)$$

When using equations (8.9) - (8.12) it is necessary to consider that the assumptions, accepted during their compilation, restrict the

field of the applicability of these equations. Them it is possible to use only at low angles of attack; they do not consider the effect of the gyroscopic effects, which appear in flight of aircraft with gas turbine engines.

For training/preparation of these equations for solution, it is necessary to determine forces and the torque/moments, which act on aircraft. From the systematic considerations of force and the torque/moments divide into the following groups:

- static forces and the torque/moments, which depend only on the flight speed and angular coordinates, i.e., acting on the aircraft, which flies without rotation/revolution;

- force and the torque/moments, which appear during the rotation/revolution of aircraft;

- control forces and torque/moments;

perturbing forces and the torque/moments, caused by the effect of medium on the flying in it aircraft.

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All these forces and torque/moments can be divided in turn, into longitudinal and lateral.

8.4. Analysis of the solution of the system of the differential equations of the disturbed motion.

The used in courses dynamics of flight the methods of the analysis of the disturbed motion are based on what investigation undergo the solutions of the system of linear differential equations. For the sake of simplicity in the recording of system of equations (8.9) and (8.10) let us lead to the following form:

$$\left.\begin{aligned}
&a'_{11}\frac{d\Delta V}{dt}+a_{11}\Delta V+a_{12}\Delta\alpha+a_{13}\Delta\vartheta=a_{10},\\
&a'_{22}\frac{d\Delta\alpha}{dt}+a'_{23}\frac{d\Delta\vartheta}{dt}+a_{21}\Delta V+a_{22}\Delta\alpha+a_{23}\Delta\vartheta=a_{20},\\
&a''_{33}\frac{d^2\Delta\vartheta}{dt^2}+a'_{32}\frac{d\Delta\alpha}{dt}+a'_{33}\frac{d\Delta\vartheta}{dt}+a_{31}\Delta V+a_{32}\Delta\alpha=a_{30},
\end{aligned}\right\} \quad (8.13)$$

$$\left.\begin{aligned}
&\frac{d\Delta\beta}{dt}+b_{11}\Delta\beta+b_{12}\omega_x+b_{13}\omega_y+b_{14}\Delta\gamma=b_{10},\\
&\frac{d\omega_x}{dt}+b_{21}\Delta\beta+b_{22}\omega_x+b_{23}\omega_y=b_{20},\\
&\frac{d\omega_y}{dt}+b_{31}\Delta\beta+b_{32}\omega_x+b_{33}\omega_y=b_{30},\\
&\frac{d\Delta\gamma}{dt}-\omega_x+b_{43}\omega_y=0.
\end{aligned}\right\} \quad (8.14)$$

The last/latter fifth equation of system (8.10) we do not record/write, since it is not required for the solution of preceding/previous four. The values of the coefficients of a_{ij} and b_{ij} are given in Table 8.1.

Systems (8.13) and (8.14) are linear nonhomogeneous equations with constant coefficients. The uniform systems of equations of $(a_{i0}=0, b_{i0}=0)$ correspond to their own airplane disturbance, when control pressed and there are no perturbation factors. The solution to equations both uniform and heterogeneous can be obtained by classical method or by operational calculus, studied in course higher the mathematicians [1].

FOOTNOTE [1]. Examples of the application/use of these methods can be found, for example, in manuals of I. V. Ostoslavskiy and I. V. Strazhevoy "flight dynamics. Stability and the controllability of flight vehicles" (M., "machine-building", 1965) and I. V. Ostoslavskogo of the "aerodynamicist of aircraft" (M., Oborongiz, 1957). ENDFOOTNOTE.

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Table 8.1. Values of the coefficients of the equations of the longitudinal and lateral disturbed motion.

a_{ij}; b_{ij}	j: 1	2	3	4	0	a'_{ij}; b'_{ij}
a_{1j}	$-R_x^V$	$-R_x^\alpha$	$-R_x^\vartheta$	0	$R_x^{\delta_\text{в}}\Delta\delta_\text{в}+R_x^{\delta_\text{н}}\Delta\delta_\text{н}+R_x^{\delta_\text{э}}\Delta\delta_\text{э}+R_x^{\delta_\text{р}}\Delta\delta_\text{р}+\Delta R_{x\,\text{в}}$	$a'_{11}=m$
a_{2j}	$-R_y^V$	$-R_y^\alpha$	$-R_y^\vartheta$	0	$R_y^{\delta_\text{в}}\Delta\delta_\text{в}+\Delta R_{y\,\text{в}}$	$a'_{22}=--mV$; $a'_{23}=mV$
a_{3j}	$-M_z^V$	$-M_z^\alpha$	0	0	$M_z^{\delta_\text{в}}\Delta\delta_\text{в}+M_z^{\delta_\text{э}}\Delta\delta_\text{р}+\Delta M_{z\,\text{в}}$	$a''_{33}=J_z$; $a'_{33}=-M_z^{\omega_z}$; $a'_{32}=-M_z^{\dot\alpha}$
b_{1j}	$-R_z^\beta/mV$	$-\alpha$	-1	$-R_z^\gamma/mV$	$\frac{R_z^{\delta_\text{н}}}{mV}\Delta\delta_\text{н}-\Delta R_{z\,\text{в}}/mV$	$b'_{11}=1$
b_{2j}	$-M_x^\beta/J_x$	$-M_x^{\omega_x}/J_x$	$-M_x^{\omega_y}/J_x$	0	$\frac{M_x^{\delta_\text{э}}}{J_x}\Delta\delta_\text{э}+\frac{M_x^{\delta_\text{н}}}{J_x}\Delta\delta_\text{н}+\frac{\Delta M_{x\,\text{в}}}{J_x}$	$b'_{22}=1$
b_{3j}	$-M_y^\beta/J_y$	$-M_y^{\omega_x}/J_y$	$-M_y^{\omega_y}/J_y$	0	$\frac{M_y^{\delta_\text{н}}}{J_y}\Delta\delta_\text{н}+\frac{M_y^{\delta_\text{э}}}{J_y}\Delta\delta_\text{э}+\frac{M_{y.\text{в}}}{J_y}$	$b'_{33}=1$
b_{4j}	0	1	$\text{tg}\,\vartheta\cos\gamma$	0		$b'_{44}=1$

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Using these methods, it is possible to obtain the solution to equations, for example system (8.13), in the form

$$\left.\begin{aligned} \Delta V &= \sum A_i e^{p_i t} + f_V(t), \\ \Delta\alpha &= \sum B_i e^{p_i t} + f_\alpha(t), \\ \Delta\vartheta &= \sum C_i e^{p_i t} + f_\vartheta(t), \end{aligned}\right\} \quad (8.15)$$

where the p_i are roots of the characteristic equation, comprised of the coefficients with unknowns in the equations of system (8.13):

$$\begin{vmatrix} a'_{11}p+a_{11} & a_{12} & a_{13} \\ a_{21} & a'_{22}p+a_{22} & a'_{23}p+a_{23} \\ a_{31} & a'_{32}p+a_{32} & a''_{33}p^2+a'_{33}p \end{vmatrix}=0. \quad (8.16)$$

the functions of $f_V(t), f_\alpha(t)$ and $f_\vartheta(t)$ in solution (8.15) are the particular solutions of system (8.13) with the right side, different from zero.

The values of A_i, B_i, C_i are integration constant, which depend on the initial conditions. Without reproducing here entire process of the search of solution, we will be restricted to the analysis of the properties of this solution.

By revealing the definition of characteristic equation (8.16), we will obtain the usual algebraic quartic equation relative to p:

$$a_0p^4+a_1p^3+a_2p^2+a_3p+a_4=0, \tag{8.17}$$

where

$$\left.\begin{aligned}
&a_0=a'_{11}a'_{22}a''_{33},\\
&a_1=a'_{11}a'_{22}a'_{33}+a'_{11}a_{22}a''_{33}-a'_{11}a'_{32}a'_{23}+a_{11}a'_{22}a''_{33},\\
&a_2=a'_{11}a_{22}a'_{33}-a'_{11}a'_{32}a_{23}-a'_{11}a_{32}a'_{23}+a_{11}a'_{22}a'_{33}+\\
&\quad+a_{11}a_{22}a''_{33}-a_{11}a'_{32}a'_{23}-a_{12}a_{21}a''_{33},\\
&a_3=a_{11}a_{22}a'_{33}-a'_{11}a_{32}a_{23}-a_{11}a_{23}a'_{32}-a_{11}a_{32}a'_{23}-\\
&\quad-a_{21}a_{12}a'_{33}+a_{21}a_{13}a'_{32}+a_{12}a_{31}a'_{23}-a_{31}a_{13}a'_{22},\\
&a_4=-a_{11}a_{32}a_{23}+a_{21}a_{13}a_{32}+a_{31}a_{12}a_{23}-a_{31}a_{22}a_{13}.
\end{aligned}\right\} \tag{8.18}$$

As can be seen from the solution of system (8.15) in of the $f_V = f_\theta = f_\varphi = 0$, of a variation in the parameters of the motions in the

course of time they can grow/rise, decrease or remain constants depending on the values of the indices of p_i which, in turn, as roots of characteristic equation (8.17), they depend on the aerodynamic and mass characteristics of aircraft.

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Since the coefficients of the characteristic equation a_0, a_1, a_2, a_3, a_4 are real values, in accordance with fundamental theorem of algebra the roots of the characteristic equation p_1, p_2, p_3, p_4 can be the real or in pairs conjugate complex numbers. If all roots of characteristic equation are real, then the dependence of variations in the parameters of motion from time bears aperiodic character and the disturbed motion it is sum four being placed one on another the particular aperiodic motions. If in this case all roots p_1, p_2, p_3, p_4 are negative, then variations in the speed, angles of attack and pitch in the course of time decrease, strive for zero with $t \to \infty$ and the disturbed motion of asymptotically stable; but if at least one of these roots is positive, then variations in the parameters of motion approach infinity with $t \to \infty$ and the disturbed motion asymptotically unstable.

If the roots of characteristic equation are the complex, mutually conjugate values, then the dependences of a variation in the parameters of motion from time bear an oscillatory nature (disturbed motion is the imposition of two oscillatory motions).

Let $p_1 = \mu + i\nu$, then $p_2 = \mu$ be $i\nu$. The oscillatory motion, described by this pair of the complex mutually conjugate roots of characteristic equation, will be that which damp or that which is being amplifying depending on the sign of real part μ complex root. If the real part of the roots is positive, then the amplitudes of a variation in the speed, angles of attack and pitch in the course of time grow/rise, strive for infinity with $t \to \infty$. In this case the oscillatory disturbed motion asymptotically unstable. If real part μ is negative, then variations in the parameters of motion in the course of time decrease, strive for zero with $t \to \infty$, and the oscillatory disturbed motion asymptotically stable. For the proof of these positions, we will use the Euler formula:

$$e^{(\mu \pm \nu i)t} = e^{\mu t} (\cos \nu t \pm i \sin \nu t).$$

the expressions, included in brackets, they are the limited values, since sine νt and cos νt cannot be more than unity. Consequently, the exponential functions of $e^{(\pm i\nu)t}$ will be those which grow/rise with the positive values of the real parts of the complex roots μ and those which decrease with negative values.

Besides those which were described above, there can be the cases, when the roots of the characteristic equation p_1 and p_2 are mutually interdependent but p_3 and p_4 - real, and vice versa, p_3 and p_4 - are mutually interdependent complex quantities, but p_1 and p_2 - real.

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The disturbed motion in these cases is imposition of two aperiodic and one oscillatory/vibratory motions.

In the case of equality to zero of real root the corresponding particular solution, as this follows from relationship (8.15), is equal to a constant value, and in equality to zero of the real part of the conjugated/combined complex roots the corresponding particular solution describes harmonic oscillatory motions. In both cases the undisturbed motion asymptotic stability does not possess, i.e., variation in the motion or their amplitude in the case of oscillatory motion strives in the process of the disturbed motion to certain constant value, different from zero. In these cases, according to A. M. Lyapunov's theorem, the examination of the linearized system does not make it possible to judge the stability of motion, and is required the study of the nonlinear system of equations of motion.

From that which was stated above it follows that for the asymptotic stability of the longitudinal undisturbed motion it is necessary and it is sufficient in order that the roots of characteristic equation would have negative real parts [1].

FOOTNOTE [1]. The real number is the special case of complex number, when the coefficient with imaginary part is equal to zero. ENDFOOTNOTE.

In the case of the characteristic equation of the n degree according to Hurwitz' theorem [2] in order that all roots of algebraic equation

$$a_0p^n+a_1p^{n-1}+a_2p^{n-2}+\ldots+a_n=0$$

would have negative real parts, it is necessary and it suffices to satisfy the following conditions:

$$\Delta_1>0,\quad \Delta_2>0,\ldots\Delta_n>0, \tag{.8 18'}$$

where

$$\Delta_1=a_1,\quad \Delta_2=\begin{vmatrix} a_1 & a_0 \\ a_3 & a_2 \end{vmatrix},$$

$$\Delta_n=\begin{vmatrix} a_1 & a_0 & 0 & 0\ldots0 \\ a_3 & a_2 & a_1 & a_0\ldots0 \\ \ldots & \ldots & \ldots & \ldots \\ a_{2n-1} & a_{2n-2} & a_{2n-3} & a_{2n-4}\ldots a_n \end{vmatrix}.$$

FOOTNOTE [2]. The proof of Hurwitz' theorem see in book Kurosh a. g. the "course of higher algebra" (M., GTTI, 1949). ENDFOOTNOTE.

For the quartic equation of condition (8.18') takes the form himself

$$a_0>0,\quad a_1>0,\quad a_2>0,\quad a_3>0,\quad a_4>0,$$
$$\Delta_3=\begin{vmatrix} a_1 & a_0 & 0 \\ a_3 & a_2 & a_1 \\ 0 & a_4 & a_3 \end{vmatrix}=a_3(a_1a_2-a_0a_3)-a_4a_1^2>0. \tag{8.19}$$

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Expression (8.19) is called routh-of Hurwitz' criterion.

After replacing in relationships (8.19) inequality signs with equal signs, we will obtain the limits of stability of the disturbed motion. Conditions (8.19) make it possible to judge the presence of the stability of the longitudinal disturbed motion, but they do not answer the question concerning the stability level or instability,

concerning the character of the disturbed motion.

Analogously is analyzed the stability of yawing motion. For the lateral disturbed motion, described by system of equations (8.14), characteristic equation will take the form

$$\begin{vmatrix} p+b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & p+b_{22} & b_{23} & 0 \\ b_{31} & b_{32} & p+b_{33} & 0 \\ 0 & -1 & b_{43} & p \end{vmatrix}=0.$$

The coefficients of b_{ij} are determined from Table 8.1. After expansion of a determinant, we will obtain

where

$$b_0 p^4 + b_1 p^3 + b_2 p^2 + b_3 p + b_4 = 0, \tag{8.20}$$

$$\begin{gathered}
b_0 = 1, \quad b_1 = b_{11} + b_{22} + b_{33}, \\
b_2 = b_{11}b_{22} - b_{12}b_{21} + b_{11}b_{33} - b_{13}b_{31} + b_{22}b_{33} - b_{23}b_{32}, \\
b_3 = b_{11}b_{22}b_{33} - b_{14}b_{21} + b_{14}b_{31}b_{43} - b_{11}b_{23}b_{32} - b_{12}b_{21}b_{33} + \\
+ b_{13}b_{21}b_{32} + b_{12}b_{23}b_{31} - b_{13}b_{22}b_{31}, \\
b_4 = [b_{23}b_{31} - b_{21}b_{33} + b_{43}(b_{22}b_{31} - b_{21}b_{32})]\, b_{14}.
\end{gathered} \tag{8.21}$$

The analysis of special cases of the longitudinal and yawing

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motions it is brought in the subsequent chapters.

PROBLEMS FOR REPETITION..

1. In what a difference of the concepts of static and transient stability?

2. Why during the expansion of functions (8.7) in Taylor series are held the only those members, who do contain variations in the kinematic parameters in degree not higher than the first?

3. What is understood by airplane disturbance?

4. With the observance of which conditions airplane disturbance can be conditionally divided into longitudinal and lateral?

Problem

satisfy the linearization of all six equations of system (8.1), i.e., lead it to form (8.6).

Chapter IX

LONGITUDINAL STATIC FORCES AND ~~TORQUE~~/MOMENTS.

9.1. The pitching moment of wing. The mean aerodynamic chord.

From the course of aerodynamics the known following relationship for determining the coefficient of the longitudinal static moment relative to the center of mass of the aircraft of the aerodynamic forces, which act on rectangular wing

$$m_{z\,\text{кр}}=c_{m0}+(\bar{x}_\text{т}-\bar{x}_F)c_y-\bar{y}_\text{т}c_{x1}. \qquad (9.1)$$

Here the c_{m0} - the moment coefficient of the wing with of $c_y=0$; the coefficient of c_{m0} is negative of concave airfoil/profiles, is equal to zero of symmetrical and is positive only of the very narrow class of S-shaped airfoil/profiles with the bent back upwards tail; $\bar{x}_\text{т}$, $\bar{y}_\text{т}$ are dimensionless $\left(\bar{x}_\text{т}=\frac{x_\text{т}}{b},\ \bar{y}_\text{т}=\frac{y_\text{т}}{b},\right.$ b - airfoil chord) the

coordinate of the center of mass of aircraft (point T in Fig. 9.1), determined in the coordinate system whose beginning is arranged on the leading edge of the cross section in question; $\bar{x}_F$ are a dimensionless longitudinal coordinate of mean aerodynamic center of wing, i.e., the point of the application/appendix of an increase in the aerodynamic force during a change in the angle of attack.

The character of the effect of the term of $\bar{y}_T c_{x1}$ on the coefficient of the pitching moment of wing profile is shown in Fig. 9.2.

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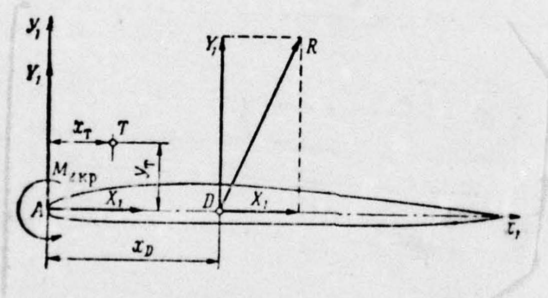

Fig. 9.1. To the determination of the pitching moment of airfoil/profile and wing of rectangular planform.

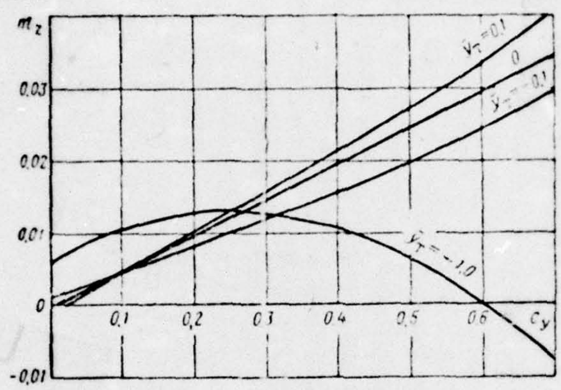

Fig. 9.2. Effect of the vertical coordinate of the center of gravity **on the coefficient** of the longitudinal the torque/moment of **airfoil/profile.**

For the majority of the contemporary aircraft of $|\bar{y}_т|<0,1,\ c_{x1}\ll c_y$, therefore the last/latter term in formula (9.1) can be disregarded in view of its smallness. Then we obtain

$$m_{z\,кр}=c_{m0}+(\bar{x}_т-\bar{x}_F)c_y. \quad (9.2)$$

With the small Mach numbers the compressibility of medium in practice barely it shows up in the value of the coefficient of the pitching moment of wing; with Mach numbers, which exceed critical [1], the coefficient of pitching moment considerably changes, which is explained by the displacement of focus to trailing wing edge (Fig. 9.3).

FOOTNOTE [1]. Critical is called the number of $M=\frac{V_\infty}{a_\infty}$, with which on the surface of airfoil/profile it appears the speed, equal to the speed of sound. ENDFOOTNOTE.

All the stability characteristics and controllability both the wing with alternating/variable on spread/scope chord and aircraft as

a whole, usually relate to the so-called mean aerodynamic chord (MAC).

The mean aerodynamic chord of the (b_A) of the wing of arbitrary planform is called the chord of this rectangular in plan/layout wing at whose area S, the pitching moment of M_z tangential X_1 and is normal Y_1 force the same as of the assigned wing. Spread/scope equivalent rectangular and assigned wings different, since in them in identical areas different chords.

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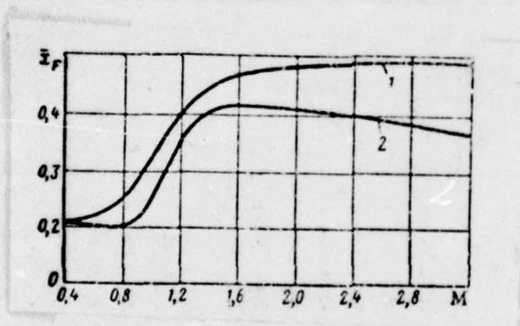

Fig. 9.3. The effect of Mach number on the position of mean aerodynamic center of wing: $\chi = 0$, $\eta = 1$, $\lambda = \infty$ (curve 1); $\chi = 0.85$ ~~is glad~~ rad, $\eta = 1$, $\lambda = 3$ (is curve 2).

Fig. 9.4. Diagram for determining the mean aerodynamic chord of wing.

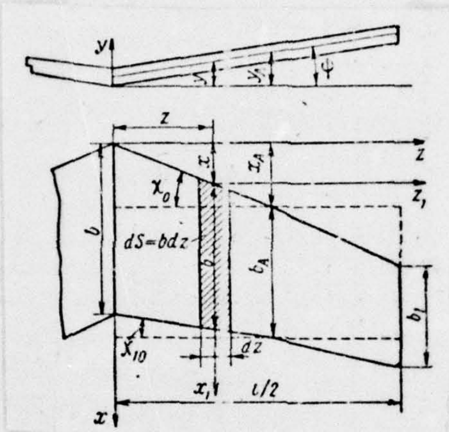

Fig. 9.4.

The value of the coefficient of the pitching moment of the wing of arbitrary planform can be determined by formula

$$m_{z\,\text{кр}}=\frac{M_z}{qSb_A}=m_{z\,0\text{кр}}+(\bar{x}_{\text{т}}-\bar{x}_F)c_y,$$

where the $m_{z\,0\text{кр}}$ - the moment coefficient of the wing with of $c_y=0$ ($m_{z\,0\text{кр}}=c_{m0}$ - for the rectangular flat/plane wing, collected from identical airfoil/profiles), $\bar{x}_F$ are a relative coordinate of mean aerodynamic center of wing).

For determining length MAC [САХ] and the coordinates of its beginning of x_A, y_A, let us compose the momental equation of the aerodynamic forces, which act on the element of area of the initial wing, relative to axle/axis Oz_1, passing through the leading edge of an airfoil profile of root cross section (Fig. 9.4):

$$dM_{z1} = q\,(c_{m0}b^2dz - c_{y1}bxdz + c_{x1}bydz), \qquad (9.3)$$

where b = b (z) - the current wing chord; x, y are coordinates of the leading edge/nose of this cell/element of wing; bdz = S - the area of the cell/element of the initial wing; (subsequently index "1" at c_{y1} is omitted, since at the low angles of the attack of $c_{y1} \approx c_y$).

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First term is the elementary moment of aerodynamic forces relative to axle/axis Oz_1, passing through the leading edge/nose of the cell/element of wing, second term is the moment of the normal force, led to the leading edge of an airfoil profile of the cross section in question relative to axle/axis Oz_1, the third term is the moment of force of periphery relative to axle/axis Oz_1.

Integrating expression (9.3) on entire wingspan, obtain the moment of aerodynamic forces relative to axle/axis z_1:

$$M_{z1} = 2q\left(\int_0^{l/2} c_m b^2 dz - \int_0^{l/2} c_y bx\,dz + \int_0^{l/2} c_{x1} by\,dz\right). \qquad (9.4)$$

the torque/moment of equivalent rectangular wing relative to the same z axis is equal to:

$$M_z = c_m^{\bullet} qSb_A - c_y^{\bullet} qSx_A + c_{x1}^{\bullet} qSy_A, \qquad (9.5)$$

where the $c_m^{\bullet}$, $c_y^{\bullet}$, $c_{x1}^{\bullet}$ are moment coefficients and aerodynamic forces of equivalent rectangular wing.

Since the torque/moment of the initial wing of arbitrary

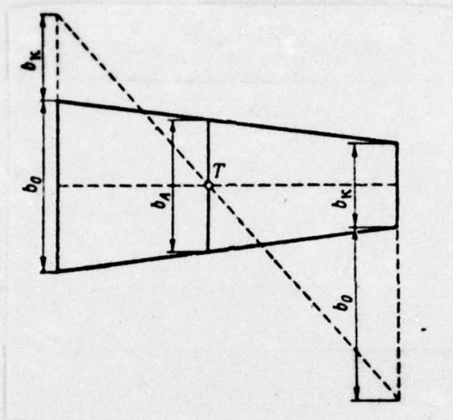
planform is equal to the torque/moment of equivalent rectangular wing, by equating to each other the right sides of relationships (9.4) and (9.5), we will obtain the following identity:

$$c_m^* qSb_A - c_y^* qSx_A + c_{x1}^* qSy_A = 2q\int_0^{l/2} c_m b^2 dz - 2q\int_0^{l/2} c_y bx dz + 2q\int_0^{l/2} c_{x1} by dz.$$

For the fulfillment of this identity it is necessary and it is sufficient in order that the corresponding terms of his right and left sides of the tale are equal. On the basis of this condition, we compose the following equalities:

$$\left.\begin{aligned} c_m^* qSb_A &= 2q\int_0^{l/2} c_m b^2 dz, \\ c_y^* qSx_A &= 2q\int_0^{l/2} c_y bx dz, \\ c_{x1}^* qSy_A &= 2q\int_0^{l/2} c_{x1} by dz. \end{aligned}\right\} \quad (9.7)$$

Fig. 9.5. Graphic method of the determination of the mean aerodynamic chord of wing.



From equalities (9.7) it is possible to determine the length of the mean aerodynamic chord of b_A and coordinate of its beginning of the x_A, y_A:

$$\left.\begin{aligned} b_A &= \frac{2}{Sc_m^*}\int_0^{l/2} c_m b^2 dz, \\ x_A &= \frac{2}{Sc_y^*}\int_0^{l/2} c_y b x dz, \\ y_A &= \frac{2}{Sc_{x1}^*}\int_0^{l/2} c_{x1} b y dz. \end{aligned}\right\} \qquad (9.8)$$

If the coefficients of pitching moment and aerodynamic forces of the initial wing are constant on entire spread/scope and are equal respectively $c_m^{\bullet}, c_y^{\bullet}, c_{x1}^{\bullet}$, that formulas (9.8) can be simplified and written in the following form:

$$\left.\begin{aligned} b_A &= \frac{2}{S}\int_0^{l/2} b^2 dz, \\ x_A &= \frac{2}{S}\int_0^{l/2} bx\,dz, \\ y_A &= \frac{2}{S}\int_0^{l/2} by\,dz. \end{aligned}\right\} \qquad (9.9)$$

In the general case for determining the length of the mean aerodynamic chord of b_A and coordinates of its beginning of x_A, y_A

it is necessary to know the laws of change in the spread/scope of the coefficients of c_m, c_y, c_{x1}, and also the chord length of the initial wing b and of the coordinate of its beginning x, y. By knowing these values, it is possible to determine aerodynamic forces and torque/moments, but then the coefficients of the c_m^*, c_y^*, c_x^* of wing equivalent are determined sufficiently simply.

In the particular case for the wing of trapezoidal planform (Fig. 9.5) when $c_m(z)=\text{const}, c_y(z)=\text{const}$ and $c_{x1}=\text{const}$, frcm formula (9.9) we will obtain

$$\left.\begin{aligned} b_A &= \frac{4}{3} b_{cp} \frac{\eta^2+\eta+1}{(\eta+1)^2}, \\ x_A &= \frac{(\eta+2)\, l \operatorname{tg} \chi_0}{6(\eta+1)}, \\ y_A &= \frac{(\eta+2)\, l \operatorname{tg} \psi}{6(\eta+1)}, \end{aligned}\right\} \quad (9.10)$$

where η - contraction, ψ - angle dihedral V.

Generally with rectilinear wing edges MAC, it passes through the center of gravity of its area.

9.2. The pitching moment of the fuselage of aircraft.

The geometric form of the fuselages of aircraft usually entirely does not correspond to the form of body of revolution. Therefore the theoretical determination of the pitching moment of the fuselage of aircraft is connected with severe difficulties. Durin; practical calculations are utilized the approximation empirical formulas, based on the generalization of the results of experiments.

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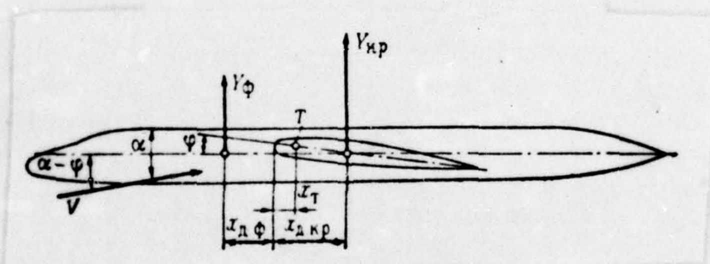

Fig. 9.6. Diagram for determining the pitching moment of fuselage.

The moment of the aerodynamic forces of fuselage relative to axle/axis Oz, passing through the center of mass of aircraft, in essence is created by the lift of Y_{Φ}. Body lift is applied in its center of pressure (Fig. 9.6) and can be calculated according to the formula whose structure is similar to the structure of formula for determining airfoil lift:

$$Y_{\Phi}=c^{\alpha}_{y\Phi}(\alpha-\varphi)S_{\Phi}q,$$

where φ is similar the angle of incidence, S_{Φ} - the area of maximum cross section of fuselage.

The pitching moment of the aerodynamic forces of fuselage relative to the center of mass of aircraft is determined by relationship

$$M_{z\,\Phi}=Y_{\Phi}(x_{\text{т}}+x_{\text{д.}\Phi})=c^{\alpha}_{y\Phi}(\alpha-\varphi)S_{\Phi}q(x_{\text{т}}+x_{\text{д.}\Phi}). \quad (9.11)$$

Expressing the $M_{z\Phi}$ through the coefficient of pitching moment

$$M_{z\,\Phi}=m_{z\,\Phi}qSb_{A} \quad (9.12)$$

and equating the right sides of equalities (9.11) and (9.12), we will obtain expression for determining the coefficient of the pitching moment of fuselage in the form

$$m_{z\,\Phi}=c^{\alpha}_{y\Phi}(\alpha-\varphi)\frac{S_{\Phi}}{S}(\bar{x}_{\text{т}}+\bar{x}_{\text{д.}\Phi}). \quad (9.13)$$

where

$$\bar{x}_{\text{н.ф}}=\frac{x_{\text{н.ф}}}{b_A},\quad \bar{x}_{\text{т}}=\frac{x_{\text{т}}}{b_A}.$$

The lift coefficient of the aircraft of c_y and angle of attack α are connected known from course aerodynamics by dependence

$$c_y = c_y^{\alpha}(\alpha - \alpha_0),$$

which can be presented, also, in this form:

$$\alpha = \frac{c_y}{c_y^{\alpha}} + \alpha_0.$$

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By substituting this value α in formula (9.13), we will obtain expressions for determining $m_{z\Phi}$ in the form of the sum of two terms:

$$m_{z\,\Phi} = c^{\alpha}_{y_\Phi}\frac{S_\Phi}{S}(\alpha_0 - \varphi)(\bar{x}_{\text{т}} + \bar{x}_{\text{д}\Phi}) + \frac{c^{\alpha}_{y\Phi}}{c^{\alpha}_{y}}\,\frac{S_\Phi}{S}\,c_y(\bar{x}_{\text{т}} + \bar{x}_{\text{д}\Phi}). \qquad (9.14)$$

First term, which does not depend on the lift coefficient of c_y and equal to the pitching moment of the aerodynamic forces of the fuselage with of $c_y=0$, let us designate by the $m_{z0\Phi}$. The value of $m_{z0\Phi}$ is negative, since zero-lift angle is usually negative or equal to zero, and the angle of incidence ϕ is always positive. Second term, proportional to lift coefficient, let us designate by the $\Delta\bar{x}_{F\Phi}c_y$. The value of $\Delta\bar{x}_{F\Phi}c_y$ is is always positive. Taking into account foregoing for the coefficient of the pitching moment of fuselage (9.14) it is possible to write

$$m_{z\Phi}=m_{z0\Phi}+\Delta\bar{x}_{F\Phi}c_y, \tag{9.15}$$

$$m_{z0\Phi}=c^{\alpha}_{y\Phi}\frac{S_{\Phi}}{S}(\alpha_0-\varphi)(\bar{x}_{\tau}+\bar{x}_{\text{л.}\Phi}); \tag{9.16}$$

where

$$\Delta\bar{x}_{F\Phi}=\frac{c^{\alpha}_{y\Phi}}{c^{\alpha}_{y}}\frac{S_{\Phi}}{S}(\bar{x}_{\tau}+\bar{x}_{\text{л.}\Phi}). \tag{9.17}$$

The value of $\Delta\bar{x}_{F\Phi}$ is the referred to b_A the distance between

the focus of the isolated/insulated wing and the focus of combination wing - fuselage. Since the center of pressure of fuselage is usually arranged in front of the center of mass of aircraft, the focus of combination wing - fuselage will be arranged in front of the focus of the isolated/insulated wing.

The value of the coefficient of the pitching moment of combination wing - fuselage not allowing for their mutual effect can be determined, after adding the piecemeal right and left sides of equations (9.2) and (9.15):

$$m_{z\,\text{кр.ф}} = m_{z\,\text{кр}} + m_{z\,\text{ф}} = c_{m0} + m_{z0\text{ф}} + (\bar{x}_{\text{т}} - \bar{x}_F - \Delta\bar{x}_{F\,\text{ф}})c_y. \qquad (9.18)$$

during the determination of the common/general/total coefficient of the pitching moment of aircraft necessary to consider the interference (mutual effect) of wing and fuselage for change in the values of $m_{z0\Phi}$ and the displacement of the mean aerodynamic center of wing cf Δx_F.

The value and the direction of the displacement of mean aerodynamic center of wing, caused by interference, depend on wing planform and Mach number of flight. During the combination of unswept wing with fuselage, its focus is displaced forward.

At subsonic rates of change in the moment coefficient of wing, caused by fuselage contribution, is so small that it can be disregarded.

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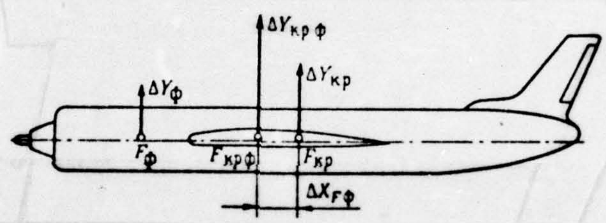

Fig. 9.7. Diagram for determining the focus of combination wing - fuselage.

Figure 9.7 shows the reason for the displacement forward of the focus of combination wing - fuselage. During a change in the angle of attack of lift increment of the wing of $\Delta Y_{\text{кр}}$ and fuselage, the ΔY_{Φ} are applied respectively at the mean aerodynamic centers of wing and fuselage at a distance of $\Delta x_{F\,\text{инт}}$, caused by interference. The focus of fuselage is located in front of wing; therefore the total lift increment of combination wing - fuselage is applied in front of mean aerodynamic center of wing. In this case, it is assumed that as a result of interference the point of the application/appendix of lift increment of wing, i.e., its own mean aerodynamic center of wing, is not displaced. For swept and deltas this assumption is incorrect.

For example in combination fuselage - swept sweptback wing (Fig. 9.8) its own mean aerodynamic center of wing is displaced back/ago.

The calculation of pitching moment and focus of combination wing - fuselage at the supersonic speeds of flow is considerably more complex; however, it is sufficiently minutely investigated both theoretically and it is experimental [1].

FOOTNOTE [1]. A. A. Lebedev, L. S. Chernobrovkin. Flight dynamics. M.,

Oborongiz, 1962. ENDFOOTNOTE.

Besides wing and fuselage in external flow, can be located engine nacelles, landing-gear fairings, suspension tanks of fuel/propellant etc. Their effect on pitching moment is considered usually only by the displacement of the longitudinal focus of aircraft, i.e., pitching moment is computed according to the formulas, similar (9.15) and (9.17):

$$m_{zi} = +\Delta\bar{x}_{Fi} c_y; \qquad (9.19)$$
$$\Delta\bar{x}_{Fi} = \frac{c^{\alpha}_{yi}}{c^{\alpha}_{y}} \frac{S_i}{S} (\bar{x}_{т} + \bar{x}_{\text{д} i}),$$

where the S_i it is computed the characteristic area of the i cell/element; $\bar{x}_{\text{д}i}$ - the coordinate of the center of pressure of the i cell/element; c^{α}_{yi} are the particular derivative of lift

coefficient the i cell/element is with respect to the angle of attack of wing.

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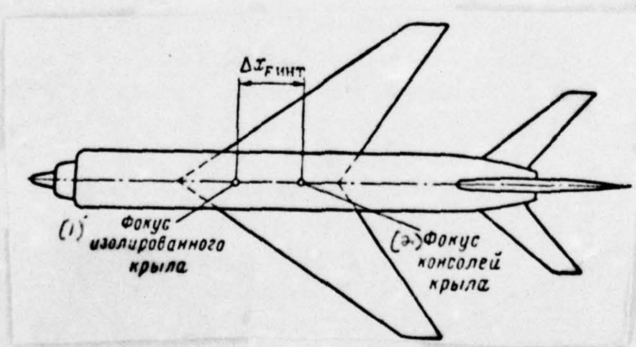

Fig. 9.8. Character of interference effect on the position of the focus of sweptback wing.

Key: (1). Focus of the isolated/insulated wing. (2). Focus of the outer planes of wing.

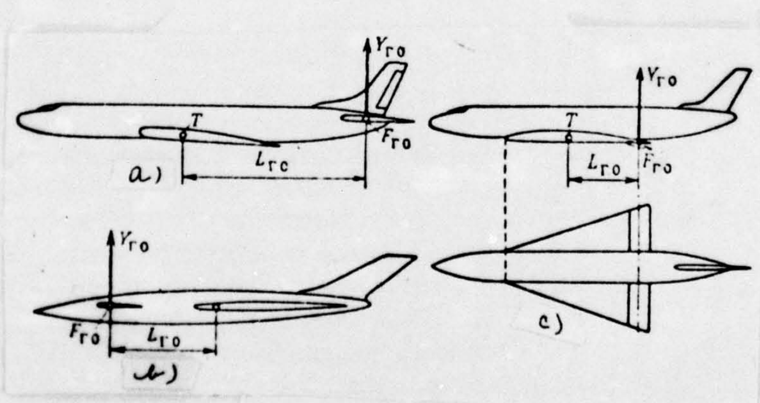

Fig. 9.9. Types of the horizontal tail assembly: a) usual diagram; b) canard configuration; c) diagram "bobtailed aircraft".

As it follows from the physical nature of the phenomenon, formula (9.20) gives the displacement of focus forward (back/ago), if the center of pressure of the protruding into flow aircraft component is arranged in front (from behind) the focus of the isolated/insulated wing.

9.3. The pitching moment of horizontal tail assembly.

Horizontal tail assembly is intended for providing for a longitudinal stability, controllability and balance of aircraft. According to the type of horizontal tail assembly contemporary aircraft can be divided into three groups (Fig. 9.9): usual diagram, canard configuration, diagram "bobtailed aircraft".

Of the aircraft of usual diagram and diagram of "weft" horizontal tail assembly can be in two versions: to consist either of fixed stabilizer and elevator or of one controllable stabilizer.

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In this paragraph is examined the torque/moment of horizontal tail assembly in the free position of controls. The moment of the aerodynamic forces of horizontal tail assembly relative to z axis, passing through the center of mass of aircraft (Fig. 9.9), is equal to:

$$M_{z\,r.o} = \pm Y_{r.o} L_{r.o}. \quad (9.21)$$

Here $Y_{r.o} \approx Y_{1r.o}$ ($Y_{1r.o}$ are normal force of horizontal tail assembly; positive sign is related to canard configuration), since at flight angles of attack the normal force of the horizontal tail assembly of $Y_{1r.o}$ with sufficient practical accuracy can be accepted to the equal lift of $Y_{r.o}$.

The arm of the horizontal tail assembly of $L_{r.o}$, strictly speaking, is equal to distance between centers of the pressure of horizontal tail assembly and the center of mass of aircraft. However, in view of a change in the center-of-pressure location with a change

of the angle of attack in the first approximation, as $L_{г.о}$, it is expedient to accept: or the distance between the focus of stabilizer and the center of mass of aircraft, if horizontal tail assembly is executed in the form of one controllable stabilizer; or the distance between centers of the suspension of control and the center of mass, if horizontal tail assembly has an elevator.

The lift of $Y_{г.о}$ is determined by relationship

$$Y_{г.о}=c_{y\,г.о}q_{г.о}S_{г.о}, \qquad (9.22)$$

where the $c_{y\,г.о}, q_{г.о}, S_{г.о}$ - respectively lift coefficient, velocity head and tailplane area.

By substituting expression (9.22) in (9.21), we will obtain

$$M_{z\,г.о}=\pm c_{y\,г.о}q_{г.о}S_{г.о}L_{г.о}. \qquad (9.23)$$

After dividing both parts of the equality into qSb_A, we will obtain expression for determining the coefficient of the pitching moment of the horizontal tail assembly:

$$m_{z\,\text{г.о}} = \pm c_{y\,\text{г.о}} k_{\text{г.о}} A_{\text{г.о}}, \tag{9.24}$$

where

$$k_{\text{г.о}} = \frac{q_{\text{г.о}}}{q}; \quad A_{\text{г.о}} = \frac{S_{\text{г.о}} L_{\text{г.о}}}{S b_A}.$$

The coefficient of $k_{г.о}$ it characterizes a decrease in the velocity in the field of horizontal tail assembly as a result of braking flow by wing and fuselage. The value of this coefficient depends on the location of horizontal tail assembly on aircraft relative the wake of wing and fuselage, and the greater the $k_{г.о}$ (nearer to unity), the more effective the application/use of a tail assembly. The greatest value of this coefficient of $(k_{г.о}=1)$ has horizontal tail assembly of the aircraft, executed by the diagram of "weft"; smallest $(k_{г.о}\approx 0,7)$ has the horizontal tail assembly of supersonic aircraft, arrange/located behind the center of mass.

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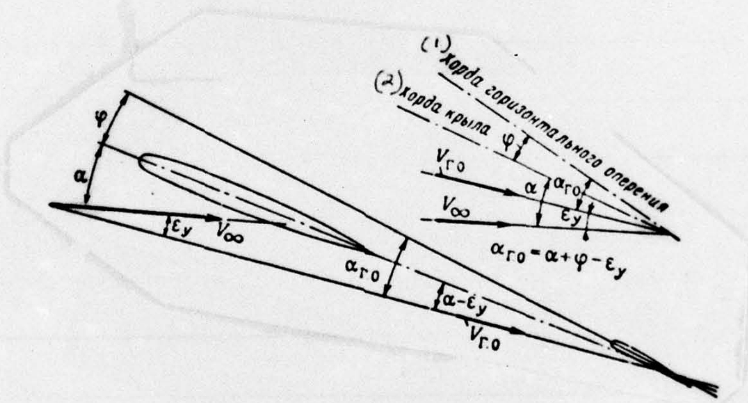


Fig. 9.10. Diagram for determining the angle of attack of horizontal tail assembly.

Key: (1). Chord of horizontal tail assembly. (2). Wing chord.

The coefficient of $A_{\Gamma.o}$ is the dimensionless area moment ratio of horizontal tail assembly relative to the center of mass of aircraft and is the important structural cell/element, which ensures longitudinal stability and aircraft handling. The value of $A_{\Gamma.o}$ of contemporary aircraft usually is located of 0.18-0.6.

On the horizontal tail assemblies of contemporary aircraft in essence, are applied symmetrical airfoil/profiles, and therefore expression for a lift coefficient it is possible to write in the form

$$c_{y\,\Gamma.o}=c_{y\,\Gamma.o}^{\alpha}\alpha_{\Gamma.o}, \qquad (9.25)$$

where the $\alpha_{\Gamma.o}$ - the angle of attack of horizontal tail assembly; $c_{y\,\Gamma.o}^{\alpha}$ they is applied derivative of the lift coefficient of horizontal tail assembly in angle of attack.

The definition of the lift coefficient of combination tail assembly - fuselage taking into account their mutual influence is conducted just as the determination of this coefficient of

combination wing - fuselage.

The angle of attack of horizontal tail assembly (Fig. 9.10) is equal to

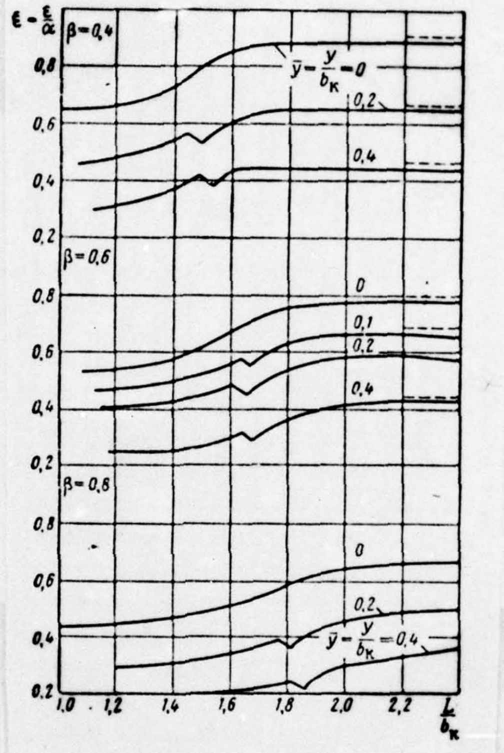
$$\alpha_{r.o}=\alpha+\varphi-\varepsilon_y, \quad (9.26)$$

where α is an angle of attack of wing; ϕ - the angle of stabilizer setting (angle between the wing chords and stabilizer); ε_y — the downwash angle of horizontal tail assembly.

For determining the downwash angle of horizontal tail assembly the greatest application/use in the calculated practice obtained the following semi-rational formula:

$$\varepsilon_y=Dc_y+\varepsilon_0. \quad (9.27)$$

Fig. 9.11. Curve/graph for determining the taper of supersonic flow after delta wing (($\beta=\sqrt{M^2-1}\,tg\,\chi$, $b_к$ are a root chord).



Here ε_0 - not depending on angle of attack downwash angle, created by fuselage; in view of smallness ε_0 it frequently they disregard; D is a coefficient of the taper of subsonic flow for the angles, in radian measure determined by relationship

$$D=\frac{0,8}{\lambda}\varkappa, \tag{9.28}$$

where λ is the effective aspect ratio of wing; $\varkappa$ - the coefficient, depending on the relative location of horizontal tail assembly and wing, or on circulation distribution according to the wingspan.

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At any point, which is located at a distance L from the leading edge/nose cf root chord and on height/altitude y from chord plane, the rake angle of supersonic flow it is possible to determine by

curve, given in Fig. 9.11 (values of the parameter β are given in the course of aerodynamics).

The rake angle of supersonic flow at horizontal tail assembly can be determined by formula (9.27), in which

$$D=\frac{\bar{\varepsilon}}{c_y^{\alpha}}, \qquad (9.29)$$

where the $\bar{\varepsilon}$ - is determined from curve/graphs in Fig. 9.11; c_y^{α} - are derivative of the lift coefficient of combination wing - fuselage in the angle of attack of wing.

By substituting the value of ε_v according to (9.27) in relationship (9.26), we will obtain expression for determining the angle of attack of horizontal tail assembly in the form

$$\alpha_{г.о}=\alpha+\varphi-Dc_y-\varepsilon_0. \qquad (9.30)$$

Taking into account formulas (9.25) and (9.30) from relationship (9.24) will obtain expression for determining the coefficient of the pitching moment of horizontal tail assembly in the free position of elevator:

$$m_{z\,r.o}=\pm c^{\alpha}_{y\,r.o}k_{r.o}A_{r.o}(\alpha+\varphi-Dc_y-\varepsilon_0). \qquad (9.31)$$

in the right side of expression (9.31) variables in the general case they are c_y, α, φ, whereupon c_y and α are connected by dependence

$$c_y=c^{\alpha}_y(\alpha-\alpha_0),$$

therefore formula (9.31) can be presented in the form

$$m_{z\,\text{г.о}} = \pm\left[c^{\alpha}_{y\,\text{г.о}} k_{\text{г.о}} A_{\text{г.о}} (\alpha_0 - \varepsilon_0) + c^{\alpha}_{y\,\text{г.о}} k_{\text{г.о}} A_{\text{г.о}} \left(\frac{1}{c^{\alpha}_{y}} - D\right) c_y + \right.$$
$$\left. + c^{\alpha}_{y\,\text{г.о}} k_{\text{г.о}} A_{\text{г.о}} \varphi\right]. \quad (9.32)$$

since into expression (9.32) for $m_{z\,\text{г.о}}$ all variables enter only to the first degree, then it it is possible to write in the form

$$m_{z\,\text{г.о}} = m_{z0\,\text{г.о}} + m^{c_y}_{z\,\text{г.о}} c_y + m^{\varphi}_{z\,\text{г.о}} \varphi, \quad (9.33)$$

where in the usual lay-out diagram of tail assembly

$$\left.\begin{aligned} m_{z0\,\text{г.о}} &= -c_{y\,\text{г.о}}^{\alpha} k_{\text{г.о}} A_{\text{г.о}} (\alpha_0 - \varepsilon_0), \\ m_{z\,\text{г.о}}^{c_y} &= -c_{y\,\text{г.о}}^{\alpha} k_{\text{г.о}} A_{\text{г.о}} \left(\frac{1}{c_y^{\alpha}} - D\right), \\ m_{z\,\text{г.о}}^{\varphi} &= -c_{y\,\text{г.о}}^{\alpha} k_{\text{г.о}} A_{\text{г.о}}. \end{aligned}\right\} \qquad (9.34)$$

9.4. Pitching moments from the thrust of engines.

On contemporary aircraft the axis of thrust of engines usually does not pass through the center of mass (Fig. 9.12). By designating through the P_x, P_y of the projection of thrust on body axes OX_1, Oy_1, let us find pitching moment from the thrust of the engines:

$$M_{zp}=x_pP_y-y_pP_x, \qquad (9.35)$$

where the x_p, y_p are arm of force of P_y and P_x the relative to the center of mass of aircraft.

The longitudinal thrust component of P_x is equal to point of tangency and is determined from Joukowski's curves. During the steady level flight it is equal to drag. Consequently, during the steady level flight the coefficient of pitching moment from the longitudinal thrust component of P_x is equal to:

$$m'_{zp} = -\frac{P_x y_p}{qSb_A} = -\frac{Xy_p}{qSb_A} = -c_x \frac{y_p}{b_A} \quad (9.36)$$

or

$$m'_{zp} = -c_x \bar{y}_p. \quad (9.37)$$

In flight angles of attack can be different; therefore the velocity vector of the motion of the center of mass of aircraft in the general case does not coincide with standard to the plane of the rotation/revolution of screw/propeller. Occurs the so-called oblique airflow, with which appears the transverse thrust component of gggg. The coefficient of pitching moment from the transverse thrust component is determined from empirical formula

$$m_{zp}^{\cdot}=0,05\frac{x_{p}iD^{2}}{Sb_{A}}c_{y},$$

where the x_p - distance from the center of mass of aircraft to the plane of the rotation/revolution of screw/propeller; i the number of screw/propellers; D - propeller diameter.

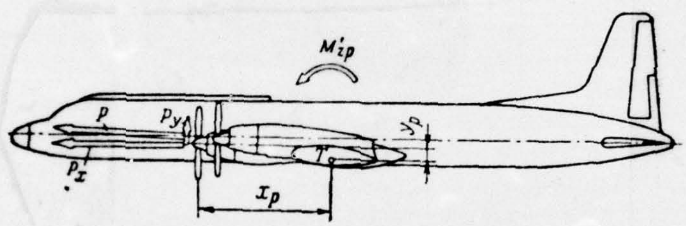

Fig. 9.12. Diagram for determining pitching moment from thrust TVD [ТВД - turboprop engine].

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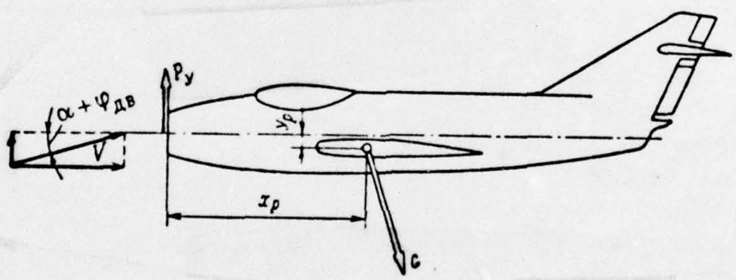

Fig. 9.13. Diagram for determining pitching moment from thrust TRD [ТРД - turbojet engine].

The transverse thrust component of jet engine is also caused by oblique airflow (Fig. 9.13) and is determined with the aid of the theorem about a change in the momentum, according to which the change in the momentum per second along the normal to the axle/axis of the engine of the equally transverse component thrust/rod:

$$P_y = m_в V \sin(\alpha \pm \varphi_д), \qquad (9.39)$$

where V is velocity of the motion of the center of mass of aircraft; α - angle of attack; $\varphi_д$ - the angle of engine installation.

The flow rate per second of the mass of the m_B of engine can be determined by formula

$$m_в = \frac{P}{W - V}, \qquad (9.40)$$

where W - the exhaust gas velocity from engine nozzle.

On the basis of formulas (9.39) and (9.40) it is possible to obtain relationship for determining transverse component thrust/rod:

$$P_y = \frac{PV \sin(\alpha \pm \varphi_\text{д})}{W - V}.$$

at low angles of attack and the angle of engine installation it is possible to place $\sin(\alpha \pm \varphi_\text{д}) \approx (\alpha \pm \varphi_\text{д})$. Then formula for determining P_Y will take the form:

$$P_y = \frac{PV(\alpha \pm \varphi_\text{д})}{W - V}.$$

pitching moment from the transverse thrust component of jet engine is equal to:

$$M^{\cdot}_{zp}=P_y x_p=\frac{PV(\alpha\pm\varphi_{\text{д}})}{W-V}x_p.$$

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After dividing both parts of the equation to qSb_A and assuming that in the steady level flight the engine thrust is equal to the drag of $P=X=c_xq,$, we will obtain expression for determining the coefficient of pitching moment from transverse component thrust/rod of jet engine in the following form:

$$m^{\cdot}_{zp}=\frac{c_x\bar{x}_p}{\frac{W}{V}-1}(\alpha\pm\varphi_{\text{д}}), \qquad (9.41)$$

where

$$\bar{x}_p = \frac{x_p}{b_A}.$$

the total moment coefficient from the engine thrust is equal to

$$m_{zp} = m'_{zp} + m''_{zp}. \quad (9.42)$$

for a propeller engine the coefficients of m'_{zp} and m''_{zp}, are determined by relationships (9.37) and (9.38), but the total coefficient of pitching moment from the thrust/rod of turbojet engine is determined by relationships (9.37) and (9.41).

The account of torque/moment from the thrust of engine has

important value during the study of stability and aircraft handling; therefore the fundamental stability characteristics and controllability must be determined in all basic engine power ratings.

In order that a change in the engine power rating would not lead to a substantial change in the coefficient of the pitching moment of aircraft, the arrangement/permutation of engines on aircraft must be so that distance from the center of mass to the thrust line bygone as possible is less. Exception/elimination can compose the case of applying a special engine for the balance of aircraft with landing and takeoff.

9.5. Longitudinal control forces and torque/moments.

Since the aircraft as absolute solid possesses six degrees of freedom, its motion can be presented as sum six motions: three forward/progressive and three rotary relative to the coordinate axes.

Control of these motions in the general case requires presence six of independent controls. However, for a man bygone very it is

difficult coordinated to realize six independent controls. In this, there is no special need: on aircraft and helicopters - the flight vehicles, which move in the dense layers of the atmosphere, controls by some motions are combined and the number of controls it does not exceed four.

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Thus, for instance, on aircraft controls of pitch and normal displacement (along the axis Oy_1) are combined in one organ/control - elevator. by one organ/control - by ailerons - is realized roll control.

Control of transverse translation (along the axis Oz_1) and by the motion of yawing is conducted with the aid of rudder, while control of the longitudinal forward motion (along the axis Ox_1) - by a power change with the aid of engine-control lever - throttle control. Control of helicopter also is realized four by organ/controls, but unlike aircraft those which were combined will be control of pitching and the longitudinal forward motion, pitch control and the longitudinal forward motion, roll control and cross

travel; those which were isolate/insulated they will be control of normal displacement (along the axis Oy_1) and by the motion of yawing.

During the action of controls, their mutual effect usually is undesirable. However, as a rule, to completely avoid this. For example on aircraft the deflection of elevator leads both to the appearance of the pitching moment and the change in the normal force and to the emergence of force of periphery, i.e., to the longitudinal translation. The deflection of rudder is accompanied by the emergence of bank, yawing and cross travel. On helicopter with tail rotor, the deflection of collective-pitch lever is accompanied by an increase in the reactionary torque, which causes the motion of yawing. For the correction of incidental motion for pilot it is necessary to deflect the appropriate control levers of ailerons - in the second; for a helicopter with tail rotor - control lever of the motion of yawing (by space of tail rotor). This mutual effect of controls requires the complex motor coordination of pilot.

The effectiveness of control is estimated at of the control force and torque/moment, created at the individual deviation of the control or control lever of it.

In the taken right-handed coordinate system positive are considered the deflections of controls counterclockwise (if we look from the side of the positive direction of the axle/axis of the body coordinate system in parallel to which is arranged the rotational axis of control). Based on this, the down-elevator deflections, and rudder will be to the right positive. For ailerons the angle of deviation is determined according to this same principle. Positive is considered the aileron deflection of the right aileron down, left - upward.

As it was noted above, as altitude controls serve elevator and the control lever of the thrust. For a control of the longitudinal forward motion, can be utilized also aerodynamic brake, reversing thrust/rod, brake parachute, etc. Along with aerodynamic controller height/altitudes for accomplishing the same functions can be utilized the stability-guidance jets (on the airplanes of vertical takeoff and landing), the thrust vector deflection of engine and other means.

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The effectiveness of control of the longitudinal forward motion in the case of use for this purpose of a power change is estimated at the derivative

$$P^{\delta_p}=\frac{\partial P}{\partial \delta_p},$$

where of the δ_p, i.e., the angle of deflection of the sector of fuel feed to engine.

Forces and torque/moments, created by the aerodynamic controller of height/altitude.

Control vane of the motion of pitch (elevator) is made either in the form of the suspended with joints being deflected part of the horizontal tail assembly or wing (elevons), or in the form of the controllable stabilizer.

If control is suspended to tail assembly or wing, then the lift of this aircraft component depends on the angles of attack and deflection of control. This dependence in the flight range of the angles of attack and deflections of control is linear:

$$\Delta Y_i = Y_i^{\alpha}\alpha_i + Y_i^{\delta_B}\delta_B, \qquad (9.43)$$

where i is the index, which shows aircraft component, to which is fastened the control; $\alpha_i=(\alpha-\alpha_0)_i$ are an aerodynamic angle of attack of this part of the airplane; α_{0i} - zero angle of attack (angle of attack with of $Y_i=0$); δ_B is an elevator angle.

Expression (9.43) can be also presented in the form

$$\Delta Y_1 = Y_i^{\alpha}(\alpha + n_B\delta_B),$$

where the $n_B=\frac{Y_i^{\delta_B}}{Y_i^{\alpha}}=\frac{c_{yi}^{\delta_B}}{c_{yi}^{\alpha}}$ - coefficient of the relative elevator-effectiveness derivative, which shows, in how often the force, which appears during the deflection of stick, more the force, created this aircraft component during a change in its angle of attack to the same value. The coefficient of the relative effectiveness of control approximately can be determined by the formulas of $n_B=\sqrt{\frac{S_B}{S_i'}}$ - at the subsonic flight speed and $n_B=\frac{S_B}{S_i'}$

- at supersonic flight speed. Here S_B - the area of elevator; S_i' - surface area, operated by control, including the area of control.

At the subsonic flight speed as a result of the deflection of elevator, occurs the change the diagram/curves of pressure in entire tail assembly (Fig. 9.14a).

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At supersonic deflection velocity of elevator changes the diagram/curve of pressure only on control (Fig. 9.14b); therefore the coefficient of the relative elevator-effectiveness derivative of gggg at supersonic speed less than at subsonic speed (Fig. 9.15) [1].

FOOTNOTE [1]. Here the expression of $\frac{m_z^\delta}{m_{z\,несж}^\delta}$ is ratio derivative taking into account compressibility to derived not allowing for compressibility. ENDFOOTNOTE.

Control force is equal to:

$$\Delta Y_{\text{в}} = Y_i^{\alpha} n_{\text{в}} \delta_{\text{в}}.$$

if x_B - the coordinate of the point of the application/appendix of control force (Fig. 9.16), then the governing torque/moment is determined from formula

$$\Delta M_{z\,\text{в}} = Y_i^{\alpha} n_{\text{в}} x_{\text{в}} \delta_{\text{в}}.$$

by the effectiveness of control accepted to call the value, equal to the torque/moment, created control during his deflection of the unit angle:

$$M_z^{\delta_{\text{в}}} = Y_i^{\alpha} n_{\text{в}} x_{\text{в}}. \quad (9.44)$$

after dividing equality (9.44) into qSb_A, will obtain expression for determining the coefficient of the elevator-effectiveness derivative:

$$m_z^{\delta_{\text{в}}} = c_{yi}^{\alpha} k_i k_{Si} \bar{x}_{\text{в}} n_{\text{в}}, \qquad (9.45)$$

where

$$k_{Si} = \frac{S'}{S};$$

the TAT of K_i - the drag coefficient of $\left(k_i = \frac{q_i}{q}\right)$.

The concrete/specific/actual expression of formula (9.45) depends on the arrangement of elevator, namely:

a) if control is established/installed on the horizontal tail assembly, arrange/located in the aft fuselage section of $L_{\text{г.о}}=-x_{\text{в}}$, (Fig. 9.16a), then

$$m_z^{\delta_{\text{в}}}=-c_{y\,\text{г.о}}^{\alpha}k_{\text{г.о}}k_{S\,\text{г.о}}\bar{L}_{\text{г.о}}n_{\text{в}}=-c_{y\,\text{г.о}}^{\alpha}k_{\text{г.о}}A_{\text{г.о}}n_{\text{в}}; \quad (9.45a)$$

b) if control is established/installed on the horizontal tail assembly, the forward facing of the fuselage (Fig. 9.16b), to $k_i=1$; $x_{\text{в}}=L_{\text{г.о}}$ and

$$m_z^{\delta_{\text{в}}}=c_{y\,\text{г.о}}^{\alpha}k_{S\,\text{г.о}}\bar{L}_{\text{г.о}}n_{\text{в}}; \quad (9.45b)$$

c) if control is arranged in the rear portion of the wing (Fig. 9.16c), of $k_{S\,\text{кр}}=\frac{S'_{\text{кр}}}{S}$; $\bar{x}_{\text{в}}=\frac{x_{\text{в}}}{B_A}$ ($S'_{\text{кр}}$ - part of the wing area, occupied with elevator), then

$$m_z^{\delta}=c_{y\,\text{кр}}^{\alpha}k_{S\,\text{кр}}\bar{x}_{\text{в}}n_{\text{в}}. \quad (9.45c)$$

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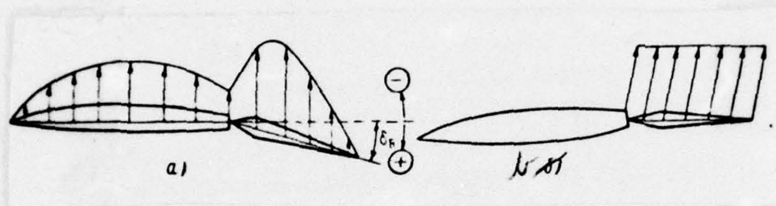

Fig. 9.14. Change in the distribution of pressure on horizontal tail assembly during the deflection of elevator in subsonic (a) and supersonic (b) flows.

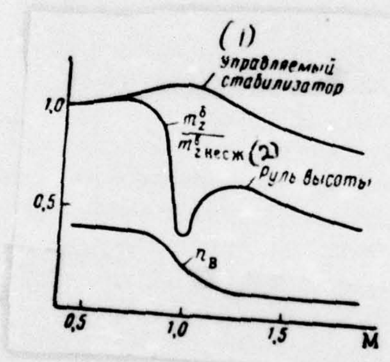

Fig. 9.15. Dependence of the elevator-effectiveness derivative on Mach number.

Key: (1). Controllable stabilizer. (2). Elevator.

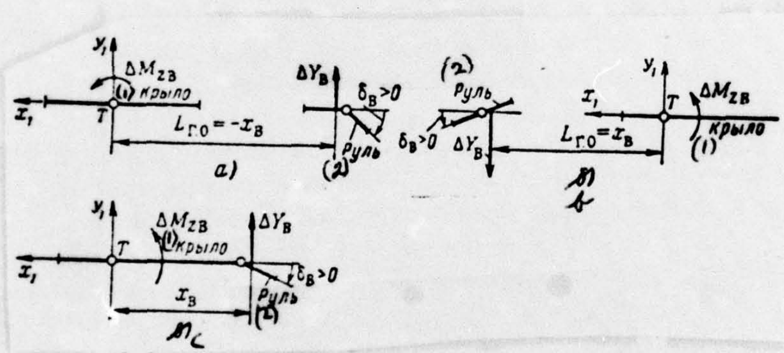

Fig. 9.16. Diagram for determining the longitudinal governing torque/moment for different airplane designs: a) usual diagram; b) canard configuration; c) diagram and "bobtailed aircraft".

Key: (1). Wing. (2). Control.

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If is deflected entire surface (controllable stabilizer, wing), then the coefficient of effectiveness is determined from formula (9.45) under the condition of $n_{\text{в}}=1$.

The obtained expressions for the coefficient of the elevator-effectiveness derivative make it possible to determine moment coefficient in the form

$$m_{z\,\text{в}}=m_z^{\delta_{\text{в}}}\delta_{\text{в}}.$$

the effectiveness of the aerodynamic controller of height/altitude as other aerodynamic characteristics, it depends on Mach number. On the subcritical Mach numbers, the coefficient of efficiency increases (increases c_y^a). The appearance of the local supersonic speeds of flow and in connection with this - local shocks which with an increase of M, being displaced back/ago, are arrange/located in the range of control, leads to a considerable

decrease in its effectiveness. Transition from transonic speeds to supersonic is accompanied by certain restoration/reduction of the effectiveness of control; however, the effectiveness of control all the same proves to be considerably less than at subsonic speeds (see Fig. 9.15).

non-aerodynamic controls of pitch.

Under conditions of the flight of aircraft at very low speed or at high altitudes the aerodynamic contrcllers are barely effective. Therefore appears the need of applying non-aerodynamic controls. As energy source for producing governing torque/moment and force in them, is utilized the energy of power plant. Let us examine the effectiveness of some of similar organ/controls.

During control of the rotation of engine (Fig. 9.17a) governing torque/moment is equal to

$$\Delta M_{zp} = -Ph_p,$$

where the h_P - the arm of force of thrust/rod relative to the center of mass of aircraft.

At the low angles of deflection of the $\delta_{\text{Д}}$ of engine

$$h_p=(x_{\text{д}}-x_{\text{т}})\sin\delta_{\text{д}}\approx(x_{\text{д}}-x_{\text{т}})\delta_{\text{д}}.$$

then the effectiveness of engine control is equal to

$$M_{zp}^{\delta_\text{л}} = P(x_\text{л} - x_\text{т}). \quad (9.46)$$

control can be carried out not only by the rotation of engine, but also by the rotation of jet nozzle and by jet vanes. In Fig. 9.17 thrust/rod of jet nozzle is designated by the P_c. and its distance of the center of gravity of L_c; Y_n is normal force of jet vane, and L_T is its distance of the center of mass.

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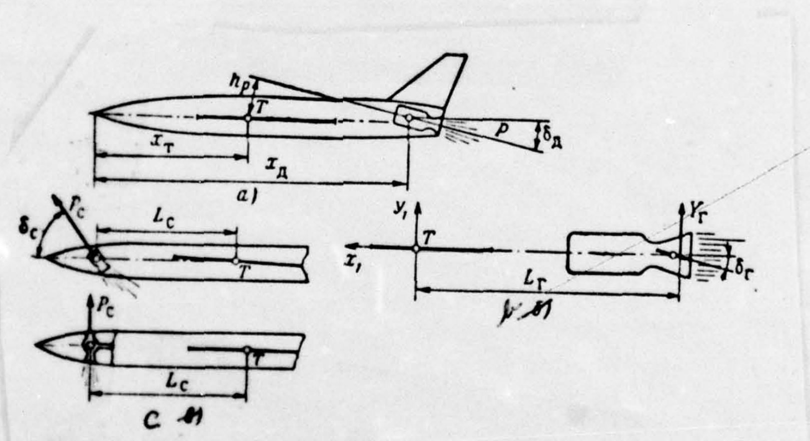

Fig. 9.17. Non-aerodynamic controls: a) control of the rotation of engine; b) jet vanes; c) stability-guidance jets.

Jet vanes - these are the actually aerodynamic controllers, established/installed in the gas jet of engine (Fig. 9.17b). The effectiveness of jet vane is similar aerodynamic:

$$M_{z\,r}^{\delta} = -Y_r^{\delta} L_r. \qquad (9.47)$$

here L_T - distance from the center of mass of aircraft to the center of pressure of jet vane;

$$Y_r^{\delta} = c_{y\,r}^{\delta} q_r S_r,$$

where the q_r are similar velocity head of gas jet; S_r - the area of jet vane.

Stability-guidance jets differ in terms of larger structural/design simplicity and widely are applied in VTOL aircraft and the apparatuses, which fly in the strongly rarefied medium. As working medium/propellant in such controls, is utilized either the gas, take/selected after turbine TRD or air, take/selected from compressor TRD, or the gas, obtained from special installation. Stability-guidance jets are made in the form of rotary or fixed nozzles (Fig. 9.17c). In the latter case the effectiveness of stability-guidance jet is determined from formula

$$M_{zc}^{\delta_c} = -\frac{\partial m_c}{\partial \delta_{m}} V_{u} L_c,$$

where the $m_{m}^{\delta_c}$ - derivative of the mass flow rate per second of air (gas) through the nozzle in terms of angle of deflection or the linear displacement of the steering control of δ_m; V_u - exhaust gas velocity; L_c - the arm of stability-guidance jet relative to the center of mass. LN3 Page 189.

For purposes of control, it can be expend/consumed to 10-12o/o of air, passing through TRD.

9.6. Coefficient of the longitudinal static moment of aircraft.

The pitching moment of entire aircraft from aerodynamic forces can be determined with the testing of model in wind tunnel of aircraft in wind tunnel. However, this method bypasses very dearly and is applied in essence in the finishing of the taken general-arrangement diagram of the design/projected aircraft. Therefore at the stage of sketch design, are applied the approximation methods of the determination of pitching moment.

The pitching moment of aircraft is equal to the sum of the pitching moments of the wing of $M_{z\,кр}$, fuselage of $M_{z\,ф}$, engine nacelles of $M_{z\,г.д}$, chassis/landing gear of $M_{z\,ш}$, horizontal tail assembly of $M_{z\,г.о}$ and, etc:

$$M_z = M_{z\,кр} + M_{z\,ф} + M_{z\,г.д} + M_{z\,ш} + M_{z\,г.о} + \ldots \qquad (9.48)$$

the mutual effect of aircraft components, as is known, it is considered during the determination of the pitching moments of aircraft components. After expressing the pitching moments of aircraft and its parts by the coefficients of pitching moments, we will cbtain

$$m_z = m_{z\,кр} + m_{z\,ф} + m_{z\,г.д} + m_{z\,ш} + m_{z\,г.о} + \ldots \quad (9.49)$$

by substituting in formula (9.49) of the value of its terms for a wing $m_{z\,кр}$ (9.2), the fuselage of $m_{z\,ф}$ (9.15), of the engine nacelles of $m_{z\,г.д}$, chassis/landing gear of $m_{z\,ш}$ (9.19), of the

horizontal tail assembly of $m_{z\,\text{г.о}}$ [(9.32) or (9.33)], the engines of m_{zp} (9.42), of elevator 9.45a), we will obtain expression for determining the coefficient of the longitudinal static moment of aircraft in the form

$$m_z = c_{m0} + (\bar{x}_\text{т} - \bar{x}_F)\,c_y + m_{z0\,\text{ф}} + \Delta\bar{x}_{F\,\text{ф}}\,c_y + \\ + \Delta\bar{x}_{F\,\text{г.л}}\,c_y + \Delta\bar{x}_{F\,\text{ш}}\,c_y - c^{\alpha}_{y\,\text{г.о}}\,k_\text{г.о}\,A_\text{г.о}(\alpha + \varphi - Dc_y - \varepsilon_0) - \\ - c^{\alpha}_{y\,\text{г.о}}\,k_\text{г.о}\,A_\text{г.о}\,n_\text{в}\,\delta_\text{р} + m'_{zp} + m''_{zp} + \ldots$$

taking into account that $\alpha_0 = \alpha_0 + \frac{c_y}{c^{\alpha}_y}$, let us write relationship (9.50) for the case of the absence of torque/moment from the engine thrust in the form

$$m_z = m_{z0} + m_z^{c_y} c_y + m_z^{\varphi}\varphi + m_z^{\delta_\text{в}}\delta_\text{в}, \qquad (9.51)$$

where

$$\left.\begin{aligned}
&m_{z0}=c_{m0}+m_{z0\,\text{ф}}-c^{\alpha}_{y\,\text{г.о}}k_{\text{г.о}}A_{\text{г.о}}(\alpha_0-\varepsilon_0),\\
&m_z^{c_y}=\bar{x}_{\text{т}}-\bar{x}_F-c^{\alpha}_{y\,\text{г.о}}k_{\text{г.о}}A_{\text{г.о}}\left(\frac{1}{c_y^{\alpha}}-D\right)+\Delta\bar{x}_{F\,\text{ф}}+\Delta\bar{x}_{F\,\text{ш}},\\
&m_z^{\varphi}=-c^{\alpha}_{y\,\text{г.о}}k_{\text{г.о}}A_{\text{г.о}}<0,\\
&m_z^{\delta_{\text{в}}}=n_{\text{в}}m_z^{\varphi}=-c_y^{\alpha}k_{\text{г.о}}a_{\text{г.о}}n_{\text{в}}<0.
\end{aligned}\right\}\quad (9.52)$$

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From formulas (9.51) and (9.52), valid at small angles of attack, it follows that the pitching moment of aircraft linearly depends on the lift coefficient (but that means and from angle of attack), of the angle of stabilizer setting and elevator angle.

If we name the longitudinal focus of aircraft point on the chord relative to which pitching moment does not depend on angle of attack, then, by designating the coordinate of this point of the aircraft through the x_{Fc}, we will obtain that with $x_{т}=x_{Fc}$, the particular product $m_z^{c_y}$ must be equal to zero. Then from appropriate equality (9.52) for the coordinate of the focus of aircraft we obtain

$$\bar{x}_{F\,c}=\bar{x}_F+c^{\alpha}_{y\,г.о}k_{г.о}A_{г.о}\left(\frac{1}{c^{\alpha}_y}-D\right)+\Delta\bar{x}_{F\,\phi}+ +\Delta\bar{x}_{F\,г.д}+\Delta\bar{x}_{F\,ш}+\ldots, \quad (9.53)$$

that it makes it possible to write the partial derivative in the form

$$m_z^{c_y}=\bar{x}_т-\bar{x}_{F\,c}.$$

of this very important relationship it follows that the particular derived $m_z^{c_y}$ is equal to the distance between centers of masses and the longitudinal focus of aircraft, divided into the mean aerodynamic chord. The sign of derivative will be negative, if the center of mass is arranged in front of the focus of $(x_T < x_F)$.

Taking into account equality (9.54) for the coefficient of pitching moment (9.51) it is possible to obtain relationship

$$m_z = m_{z0} + (\bar{x}_T - \bar{x}_{F\,c}) c_y + m_z^{\varphi} \varphi + m_z^{\delta_n} \delta_n. \qquad (9.55)$$

the analysis of the longitudinal static stability and aircraft handling can be made with the aid of equation (9.55) in which all coefficients with of alternating/variable c_y, φ, δ_B and the absolute term of m_{z0} are determined from relationships (9.52) and (9.54).

Questions for repetition.

1. Explain, why the focus of the isolated/insulated wing under the effect of the fuselage with of $M<M_{кр}$ is displaced to the spout of the mean aerodynamic chord?

2. To which side from the focus of the isolated/insulated wing, is displaced the mean aerodynamic center of wing taking into account the effect of engine nacelles, arrange/located on aft fuselage section? Why?

3. Can be the coefficient of $k_{r.o}>1$ for aircraft with engine-propeller combination?

4. How do change the torque/moments of horizontal tail assembly as a result of downwash, created with wing?

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5. To an increase in which torque/moment - pitching or diving does lead the oblique airflow of the screw/propellers of aircraft?

6. Examine effect on the effectiveness of the control of Mach number, of the angle of attack and geometric dimensions of horizontal tail assembly.

The problem

to determine the displacement of the mean aerodynamic center of wing of aircraft Il-18, caused by the nose effect of the fuselage, if it is known that the center of pressure of its is arranged at a distance 5 m of the leading edge/nose of fuselage, the mean aerodynamic chord is equal to 4.05 m and its leading edge is arranged in 14 m of the leading edge/nose of fuselage. Are known also the following data: the derivative of the lift coefficient of the forward fuselage in the angle of attack of $c^{\alpha}_{y\,нос}=2$ (is related to maximum cross section of fuselage), the same wing of $c^{\alpha}_{y}=5{,}2,$ the diameter of the midsection of fuselage 3.5 m, the wing area 140 m², the center of gravity are arranged at a distance 20o/o MAC.

Answer/response; $\Delta\bar{x}_{F\phi}=0{,}064.$

end section.

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Chapter X

LATERAL STATIC FORCES AND ~~TORQUE~~/MOMENTS.

10.1. Lateral forces.

In flight of aircraft without slip, it will flow itself by symmetrical flow relative to its plane of geometric symmetry; therefore pressures on the lateral surfaces of aircraft are symmetrically distributed relative to this plane. In flight with slip, the symmetry of flow is disturbed, in consequence of which on the lee side pressure is reduced, and on windward - it is raised. The pressure difference is the reason for the emergence of the lateral aerodynamic force of Z, arrange/located in plane perpendicular to velocity vector. At glancing angles, the lateral force is virtually equal transverse - resulting the forces of pressure, directed along the normal to the plane of symmetry.

The approximate character of the distribution of the forces of pressure, which act on the lateral surfaces of aircraft at subsonic and supersonic flight speeds, is shown in Fig. 10.1.

The dominant role in producing the lateral aerodynamic force belongs to fuselage and the vertical tail assembly of aircraft. Can turn out to be also considerable the force, caused by the unsymmetric flow about the engine nacelles or packages of engine plants. As

concerns wing, then its portion/fraction in producing the lateral force is insignificant, and usually it they disregard.

The node of the action of the resultant of the lateral forces Z with the plane of symmetry (point of F_β in Fig. 10.2) is called of the lateral center of pressure of airplane; on the strength of geometric symmetry the center of pressure and focus coincide.

Thus, the lateral aerodynamic force can be represented in the form of sum

$$Z_1 \approx Z = Z_{\Phi} + Z_{\text{в.о}} + Z_{\text{м.г}}, \qquad (10.1)$$

where the $Z_{\Phi}, Z_{\text{в.о}}, Z_{\text{м.г}}$ are the lateral forces, created respectively by fuselage, by vertical tail assembly and engine nacelles. Since in flight slip angle does not exceed 0.1-0.15 is glad, in this range of slip angles dependence Z = f (β) can be accepted linear; therefore

$$Z = Z^{\beta}\beta = c_z^{\beta} qS\beta. \qquad (10.2)$$

Fig. 10.1. Approximate distribution of forces of pressure on the lateral surfaces of aircraft during subsonic (a) and supersonic flight speeds (b).

Fig. 10.2. Diagram of the lateral forces, which act on aircraft.

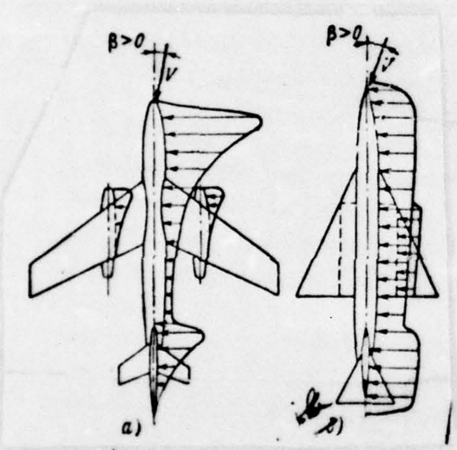

Fig. 10.1

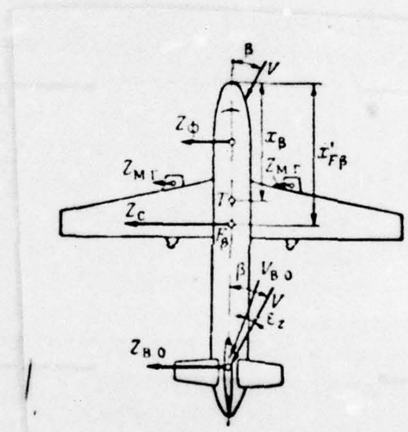

Fig. 10.2

In the adopted system of coordinates, the slip angle is considered positive, if slip is made for the right wing (see Fig. 10.1). With β > 0 lateral force and, consequently Z^{β} they will be negative.

The lateral force, created by fuselage, is determined analogous with the determination of its lift:

$$Z_{\Phi}=c^{\beta}_{z\,\Phi}qS_{\Phi}\beta. \quad (10.3)$$

derivative of side-force coefficient in slip angle $c^{\beta}_{z\Phi}$ is approximately equal $c^{\alpha}_{y\Phi}$.

With slip, as a result of the taper of vortex wake after wings to the side of the delaying wing and effect of fuselage on flow, slip angle in the field of vertical tail assembly it differs from angle β

undisturbed flow to the angle of the lateral taper of ε_z. Taking this into account of influence, the vertical tail lift can be determined by formula

$$Z_{\text{в.о}}=c_{z\,\text{в.о}}^{\beta}q_{\text{в.о}}S_{\text{в.о}}(\beta-\varepsilon_z),$$

where the $q_{\text{в.о}}$ - velocity head in the field of vertical tail assembly.

The lateral force, created by engine nacelles, is determined analogous with the lateral force of fuselage.

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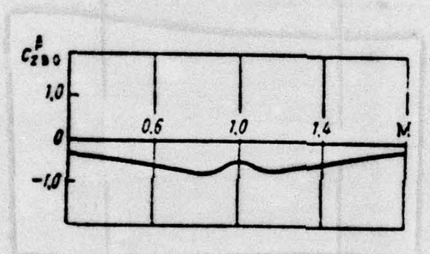

Fig. 10.3. Typical graph/diagram of the dependence of $c^{\beta}_{z\,\text{в.о}}=f(M)$.

The side-force coefficient of aircraft according to formulas (10.1) - (10.4) can be presented in the form

$$c_z = c_z^{\beta}\beta = \left[c^{\beta}_{z\,\Phi} k_{S\,\Phi} + c^{\beta}_{z\,\text{в.о}} k_{\text{в.о}} k_{S\,\text{в.о}} \left(1 - \frac{\partial \varepsilon_z}{\partial \beta}\right) + \right.$$
$$\left. + c^{\beta}_{z\,\text{м.г}} k_{S\,\text{м.г}}\right]\beta, \qquad (10.5)$$

where the $k_{Si} = \frac{S_i}{S}$ are a relative area of the i aircraft component; $K_{B.O} = \frac{q_{B.O}}{q}$ - the drag coefficient of flow in the field of vertical tail assembly.

The value of $k_{\text{в.о}}$ depends on the place of the arrangement of vertical tail assembly on aircraft and Mach number. With $M < M_{\text{кр}}$ the value of $k_{\text{в.о}}$ lie/rests at limits of 0.9-0.95, with M > 1.3 is equal to unity. Approximate dependence $c^{\beta}_{z\text{в.о}} = f(M)$ is given in Fig. 10.3.

The lateral force is created by engine plant. With slip the flow

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falls in input device TRD ['TPД - turbojet engine] or approaches the plane of the rotation/revolution of screw/propeller at an angle the axle/axis of engine, by approximately equal to β, which is accompanied by a change in the momentum/impulse/pulse of flow along the normal to axle/axis; as a result appears the transverse force, which acts on screw/propeller or the lateral surface of input device TRD. This force it is possible to define just as the transverse force of power plant, caused by angle of attack.

Of some layouts of aircraft, exhaust jet or air jet, reject/thrown by screw/propeller, changes the lateral rake angle in the field of tail assembly and thereby the value of the lateral force, created to them. This effect more reliably can be taken into account on the base of experimental studies.

10.2. The static moment of yawing (turning up torque/moment).

By the yawing moment, or by the turning up torque/moment, it is accepted to call moment with respect to body axis Oy_1. The yawing moment is created by the lateral force of aircraft. Since with the lateral forces we were acquainted above, it remains to determine the

points of their application/appendix, i.e., the lateral foci of loose parts and aircraft as a whole.

With the subcritical Mach numbers, the center of pressure (focus) of the isolated/insulated fuselage and engine nacelles is arranged approximately at a distance of $(0,1\div0,15)l_{\Phi}$ from its leading edge/nose and virtually does not change to $M=M_{\text{кр. ф}}$.

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Transition to supersonic flight speeds leads to the qualitative change in the distribution of the lateral forces of pressure along fuselage (see Fig. 10.1) and to the center-of-pressure travel back/ago. Furthermore, on the center-of-pressure location substantially affects the form of the forward fuselage, depending on which at supersonic speeds the center of pressure can turn out to be that which was arrange/located at a distance of $(0,20\div0,6)l_{\Phi}$. With an increase in the slip angle, the center of pressure of fuselage is displaced back/ago. In the first approximation, with low β this effect can be disregarded.

The center of pressure (focus) of vertical tail assembly is defined just as mean aerodynamic center of wing or horizontal tail assembly (chapter IX).

By analogy with pitching moment the aerodynamic static moment of yawing of the M_y of aircraft with the hcrizontal arrangement of wings can be determined on the one hand by formula (Fig. 10.2):

$$M_y = -Z(x_T - x_{F\vartheta}),$$

with another - according to formula from experimental aerodynamics:

$$M_y = m_y S \frac{\rho V^2}{2} l.$$

equating the right sides of the expressions for M_y and after expressing Z by the coefficient of c_z^β in accordance with (10.2),

we will obtain

$$m_y = -c_z^\beta(\bar{x}_T - \bar{x}_{F\beta})\beta,$$

where the $\bar{x}_T, \bar{x}_{F\beta}$ - respectively the coordinate of the center of mass and lateral focus of aircraft, expressed in the portion/fractions of the spread/scope of wings.

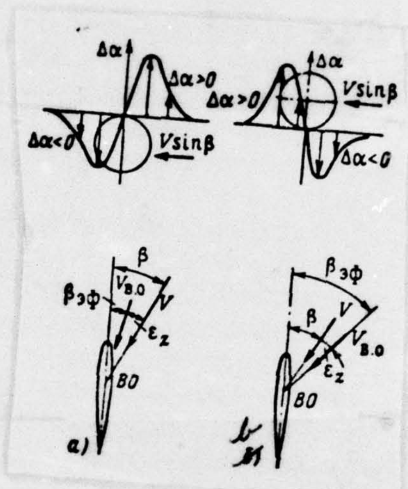
The coordinate of the lateral focus of $\bar{x}_{F\beta}$ as point of the application/appendix of the resultant of all lateral forces can be expressed as

$$\bar{x}_{F\beta} = \frac{1}{c_z^\beta}\sum c_{zi}^\beta k_{Si}\bar{x}_{Fi}k_i(1-\varepsilon_z^\beta), \quad (10.7)$$

where the $\bar{x}_{Fi}$ - the coordinate of the foci of fuselage, vertical tail assembly and engine nacelle, expressed in the portion/fractions of the spread/scope of wings; $\varepsilon_z^\beta = \frac{\partial\varepsilon_z}{\partial\beta}$ are derivative of the lateral

rake angle in terms of slip angle. Considerable influence on the value of ε_z^β exerts wing arrangement by height of fuselage. In diagrams "high wing monoplane" and "low wing monoplane", most widely accepted for transport aircraft, this influence most is substantial; $k_i=\frac{q_i}{q}$ - the drag coefficient of flow in the field of the i aircraft component in question.

Fig. 10.4. Interference effect of wing and fuselage on the angle of the lateral taper in the field of the tail assembly: a) high wing monoplane; b) low wing monoplane.



In the case of flight in a slip, the interaction of wing and fuselage in diagram "high wing monoplane" is equivalent to an increase in the effective angle of attack in the cross sections, close to on-board, the pushed forward in flow wing and its decrease in the same cross sections of the delaying wing (Fig. 10.4). This unsymmetric distribution of supplementary angles of attack (that means and $c_{y\,\text{сеч}}$) leads to the appearance of the lateral pressure gradients, which produce downwash to the side of the pushed forward wing in the points, arrange/located higher than the wing plane, and to the side of the delaying wing at the points, arrange/located lower than the wing plane. The vertical tail assembly usually is arrange/located above wing plane; therefore the phenomenon described above lowers the effective angle of the vertical tail assembly of $\beta_{\text{эф}}$ (Fig. 10.4a). In this case, decreases the torque/moment, created by vertical tail assembly, which produces the need for the setting up of more powerful vertical tail assembly on the aircraft, executed by diagram "high wing monoplane".

In diagram the "low wing monoplane" the interaction of wing and fuselage with slip leads to inverse effect and, therefore, to an increase in the effective angle of vertical tail assembly (Fig. 10.4b).

The yawing moment, created by thrust of engines, in essence is determined by torque/moment from the transverse thrust component. If the coordinate of the entrance of the engine of $x_{\text{д.вх}}$, then the yawing moment, created thrust relative to the center of mass, will be equal to

$$M_{yp} = P_z x_{\text{д.вх}}.$$

One should consider the influence of the supplementary torque/moment, caused by exhaust jet TRD or by air jet behind screw/propeller, to air-load distribution according to vertical tail assembly and aft fuselage section.

10.3. The static moment of the bank of aircraft.

By the moment of roll it is accepted to call moment with respect to body axis Ox_1 aircraft. The static moment of the bank of aircraft in essence is created by wing and vertical tail assembly.

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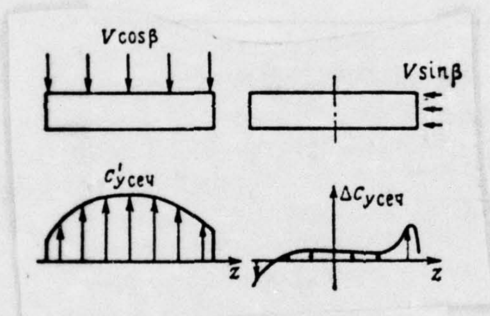

Fig. 10.5. Diagram of flow around of the wing with slip.

Moment of roll, created by wing. The flow of asymmetric flow about the rectangular wing can be presented as result of the imposition of two flows: normal to leading wing edge and spanwise flow (Fig. 10.5).

distribution pressure from wing section it is determined normal by velocity component V cos β, therefore, if we disregard the wing-tip effect, lift it varies in proportion to $\cos^2\beta$.

The lateral flow as a result of the finiteness of the wingspan exerts a substantial influence on the character of wing loading, especially in its end cross sections. The effect of this factor grow/rises with a decrease in the wing aspect ratio.

In the first approximation, yaw effect on pressure distribution according to the wing sections of the final spread/scope can be explained by the summation of the pressures, caused by flows with a velocity of V cos β and V sine β. For a flow with a velocity of V sine β by leading wing edge serves the end of the pushed forward wing, and rear, i.e., the end of the delaying wing. The distribution of $c_{y\,\text{сеч}}$ according to the spread/scope of plane the rectangular

pryamougol'go of wing in this case becomes unsymmetric.

The unsymmetric distribution of $c_{y\,\text{сеч}}$ is the reason for the appearance of the heeling torque/moment, directed to the side of the delaying wing. Of tapered wing with an increase in the contraction, the effect of end effect decreases and, therefore, decreases the value of the heeling torque/moment, and with η = 2.5-3 can be changed its even sign.

Effective means for a change of the heeling torque/moment in the necessary direction is the change of dihedral arm sweep of (ψ) - the angle between axle/axis Oz_1 and the projection of leading wing edge on plane y_1Oz_1. The angle of ψ is considered positive, if end wing sections are arranged above root cross sections.

The effect of dihedral arm sweep on load distribution according to yawing wing can be explained, if we present shear flow in the form of the sum of two flows, as this is bygone made above. The lateral flow with a velocity of V sine β leads to an increase in the angles of attack in the cross sections of the pushed forward wing at the positive of dihedral arm sweep (Fig. 10.6) and to a decrease in them

in the cross sections of the delaying wing by the value, determined by the following expression:

$$\Delta\alpha = \frac{\sin\beta \sin\psi}{\cos\beta} = \operatorname{tg}\beta \sin\psi.$$

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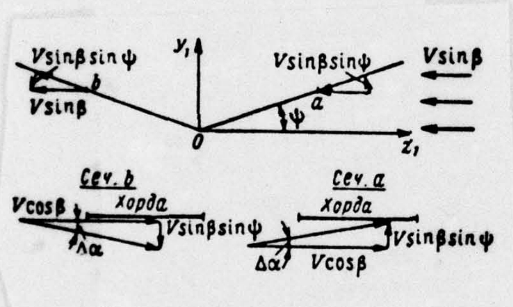

Fig. 10.6. Diagram of yawing wing with angle dihedral.

Since in flight the slip angles do not exceed 0.2 is glad, and of dihedral arm sweep not more than 0.1 is glad, change in the angle of attack of wing sections approximately equally

$$\Delta\alpha \approx \pm \beta\psi, \qquad (10.8)$$

whereupon positive sign is related to pushed forward, but "minus" is related to the delaying wing.

This unsymmetric distribution of supplementary angles of attack according to wings causes the unsymmetric increase of $c_{y\,сеч}$ and - as result of this - the emergence of the heeling torque/moment, directed to the side of the delaying wing. The value of this torque/moment is determined by integration for the spread/scope of the torque/moment, created by an increase in the normal force, which acts on the element of area of wing bdz:

$$M_{x\psi} = -2q\int_0^{l/2} c_y^{\alpha} |\Delta\alpha| bz\,dz.$$

if we accept $c^{a}_{y\,\text{сеч}}$ as constant for spread/scope, then after integration and division of the obtained result for qSl for a tapered wing we will obtain

$$m_{x\psi} = -0,166\,\frac{\eta+2}{\eta+1}\,c^{a}_{y}\beta\psi, \qquad (10.9)$$

where the $m_{x\psi}$ - the coefficient of the heeling torque/moment, caused of dihedral arm sweep, η - wing taper.

It should be noted that the torque/moment, caused by the

presence of dihedral arm sweep, does not depend on angle of attack (we are speaking about angles of attack, where the dependence of $c_y=f(\alpha)$ linear).

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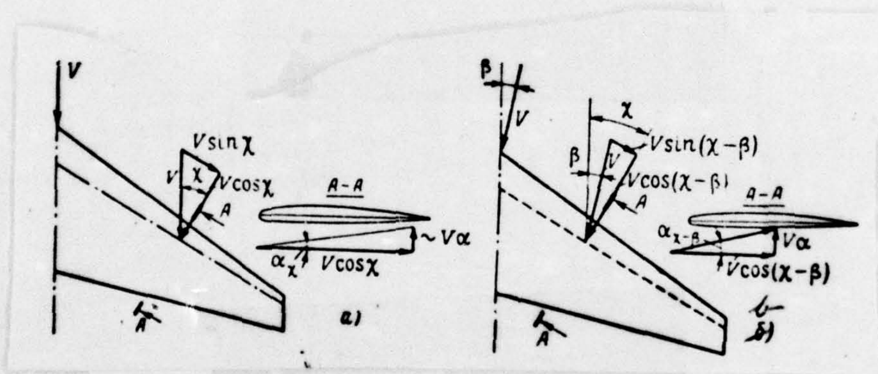


Fig. 10.7. Effective angle of attack of sweptback wing with $\beta = 0$ (a) and $\beta \neq 0$ (b).

In flight with slip to aircraft with sweptback wing, acts the supplementary heeling torque/moment, caused by sweepback.

The appearance of this torque/moment is caused by different effective slip angles of the pushed forward along flow wing and of that which delay:

$$\beta_{\text{эф}} = \chi \pm \beta,$$

where χ is a sweep angle, positive sign is related to that which delay, and "minus" to the pushed forward wing.

Of sweptback wing the effective angle of attack of wing (angle chordwise, normal to the line of 1/4 chords, Fig. 10.7a) will be equal to:

$$\alpha_{\chi} \approx \frac{V\alpha}{V\cos\chi} = \frac{\alpha}{\cos\chi}.$$

with slip (Fig. 10.7b):

for the pushed forward wing

$$\alpha_{\text{выд}} \approx \frac{\alpha}{\cos(\chi-\beta)};$$

for the delaying wing

$$\alpha_{\text{отс}} \approx \frac{\alpha}{\cos(\chi+\beta)}.$$

the lift, created by the cell/element of wing dz, is equal to:

for the pushed forward wing

$$dY_{\text{выд}} = c^{\alpha}_{y\chi} qb\alpha \frac{\cos(\chi - \beta)}{\cos \chi} dz;$$

for the delaying wing

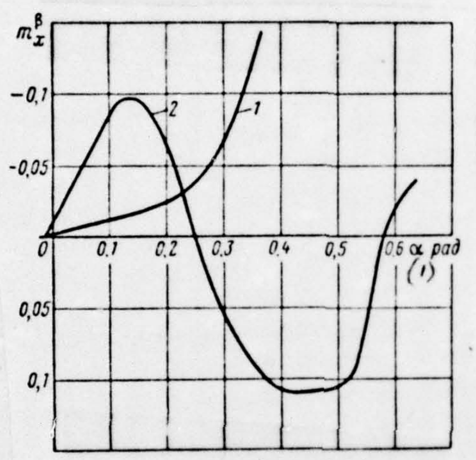
$$dY_{\text{отс}} = c^{\alpha}_{y\chi} qb\alpha \frac{\cos(\chi + \beta)}{\cos \chi} dz,$$

where

$$c^{a}_{y\chi} \approx c^{a}_{y\chi=0}\cos\chi.$$

Fig. 10.8. Approximate experimental dependence of $m_x^\beta = f(\alpha)$ for unswept (1) and swept (2) wings.

Key: (1). is glad.



After determining the elementary torque/moment, created by these forces of relatively longitudinal axis, after integration for spread/scope with low β and the constant of $c_{y\chi}^{\alpha}$ we will obtain the following expression for the coefficient of the heeling torque/moment, caused by the sweepback:

$$m_{x\chi} = -0,166 \frac{\eta+2}{\eta+1} c_y \beta \operatorname{tg} \chi. \qquad (10.10)$$

comparing dependences (10.9) and (10.10), it is possible to make the conclusion that the rolling-moment coefficient, caused by sweepback, unlike moment coefficient, caused of dihedral arm sweep, depends on lift coefficient (angle of attack) and with an increase in the latter it increases in absolute value.

However, it is necessary to remember that the discussion concerns flight angles of attack (α < 0.1-0.2 is glad); at high angles of attack, as a result of the peculiar development of

disruption/separation on the spread/scope of sweptback wing, the coefficient of the heeling torque/moment, caused by sweepback, with an increase in the angle of attack, it will decrease in absolute value and, beginning with certain angle of attack, it can change sign to opposite. Of unswept wing such phenomena are not observed (Fig. 10.8).

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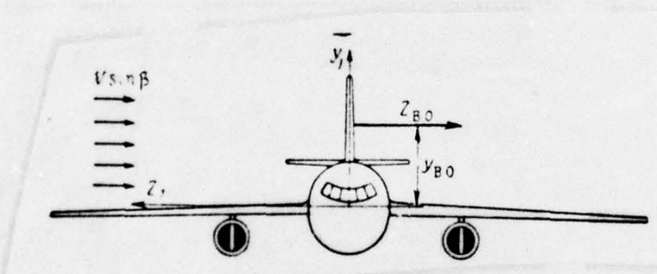

Fig. 10.9. Moment of roll, created by vertical tail assembly.

Thus, the rolling-moment coefficient of wing in the general case is equal to

$$m_{x\,\text{кр}}=m_{x0}+m_{x\psi}+m_{x\chi}, \qquad (10.11)$$

where the $m_{x\delta}$ - moment coefficient, proportional to slip angle, caused by end effects; $m_{x\psi}$, $m_{x\chi}$ are the moment coefficients, which depend respectively on angle dihedral and the sweepback of wing.

Moment of roll, created by vertical tail assembly. The vertical tail lift, caused by slip, while is applied above longitudinal axis of aircraft (Fig. 10.9), creates the heeling torque/moment, equal to

$$M_{x\,\text{в.о}}=c^{\beta}_{z\,\text{в.о}}q_{\text{в.о}}S_{\text{в.о}}y_{\text{в.о}}(\beta-\varepsilon_z),$$

coefficient of which it is expressed by formula

$$m_{x\,\text{в.о}}=c_{z\,\text{в.о}}^{\beta}\,k_{\text{в.о}}k_{S\,\text{в.о}}\,\bar{y}_{\text{в.о}}\,(1-\varepsilon_z^{\beta})\,\beta=m_{x\,\text{в.о}}^{\beta}\beta,$$

where the $\bar{y}_{\text{в.о}}=\frac{y_{\text{в.о}}}{l}$ are a dimensionless coordinate of the center of pressure of vertical tail assembly.

Moment of roll, caused by wing-root interference effect. As it was noted above, with slip depending on wing arrangement by height of fuselage, occurs the redistribution of load on the wingspan.

In the case of diagram "high wing monoplane" of increase, the $c_{y\,\text{сеч}}$ bear the unsymmetric character: positive increase on pushed forward and negative during the delaying wing, which produces the supplementary moment of roll, which acts to the side of the delaying wing. At glancing angles the rolling-moment coefficient directly of proportional to angle β:

$$m_{x\,\text{инт}}=m_{x\,\text{инт}}^{\beta}\beta. \qquad (10.13)$$

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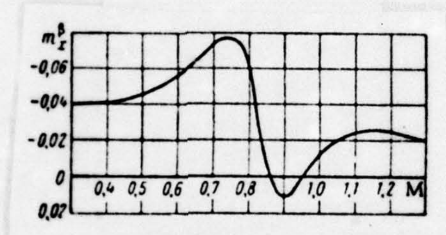

Fig. 10.10. The approximate dependence of $m_x^\beta = f(M)$ ($\chi = 0.6$ is glad, $\psi = 0.05$ is glad, $\alpha = 0.04$ is glad).

In the case of diagram "low wing monoplane" the picture is reverse/inverse: the supplementary moment of roll acts to the side of the pushed forward along flow wing. With the average/mean wing arrangement, supplementary torque/moment virtually is absent.

Thus, the coefficient of the static moment of the bank of aircraft can be determined in the form of the sum:

$$m_x = m_x^\beta \beta = m_{x\,\text{кр}} + m_{x\,\text{в.о}} + m_{x\,\text{инт}}. \qquad (10.14)$$

the rolling-moment coefficient of aircraft depends on Mach number, which is connected with a change in the lift effectiveness of aircraft components in M. The approximate dependence of $m_x^\beta = f(M)$ is given in Fig. 10.10.

10.4. Lateral control forces and torque/moments.

The lateral aircraft control is realized by two organ/controls: a rudder and by the ailerons, intended for a change in the yaw angles, slip and bank.

The functions of these organ/controls are interconnected: the deflection of rudder leads to the rotation of the nose of aircraft and to bank, but during the aileron deflection along with the moment cf roll appears the yawing moment. For the target/purpose of a decrease in the effect of ailerons on the motion of yawing on some aircraft, are applied differential ailerons. Of such ailerons the angles of deflection upward and downward different (it is upward more, but it is down less), which makes it possible to virtually eliminate yawing during the aileron deflection. On high-speed aircraft instead of the ailerons either along with ailerons they are applied the lamellar (Fig. 10.11a) or jet/stream (Fig. 10.11b) interceptors, which produce boundary-layer separation on suction side of wing, in consequence of which normal aerodynamic force in cross sections with the pushed forward interceptor decreases, and tangential increases.

Fig. 10.11. Diagram of the action of lamellar (a) and jet/stream (b) interceptor/spoilers.

Fig. 10.12. Diagram for determining the governing torque/moment, created by rudder.

Key: (1). Vertical tail assembly.

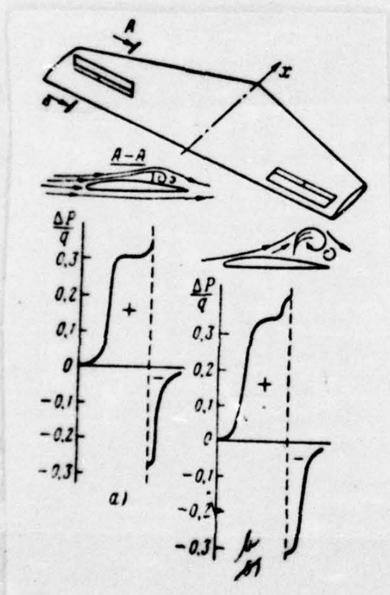

Fig. 10.11

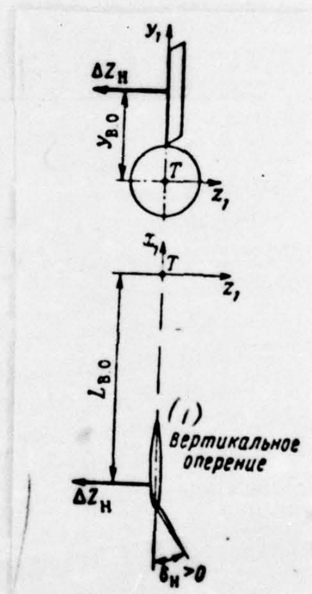

Fig. 10.12

For the aircraft, which use states of motion at very low speeds or at high altitudes, when aerodynamic controllers are ineffective, just as in the case of pitch control, are applied the jet/stream and jet vanes or the control, realized by the rotation of the thrust.

Forces and the torque/moments, created by rudder. During the deflection of rudder, appears the lateral force, which acts to the side, opposite to the rotation of control (Fig. 10.12). This force creates moments with respect to axle/axes Ox_1 and Oy_1, which are respectively equal to:

$$\left.\begin{aligned} \Delta M_{x\,\text{н}} &= \Delta Z_{\text{н}} y_{\text{в.о}} = c^{\delta}_{z\,\text{в.о}} q_{\text{в.о}} S_{\text{в.о}} y_{\text{в.о}} \delta_{\text{н}}, \\ \Delta M_{y\,\text{н}} &= \Delta Z_{\text{н}} L_{\text{в.о}} = c^{\delta}_{z\,\text{в.о}} q_{\text{в.о}} S_{\text{в.о}} L_{\text{в.о}} \delta_{\text{н}}, \end{aligned}\right\} \quad (10.15)$$

where the δ_H, the rudder angle,, which is considered positive during the rotation of control to the right.

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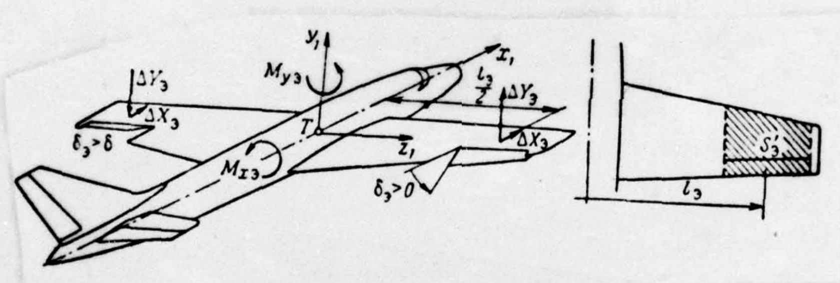

Fig. 10.13. Diagram of forces and torque/moments, which act on aircraft during the aileron deflection.

The coefficient of the rudder-effectiveness derivative we find with the aid of formulas (10.15):

$$\left.\begin{aligned} m_x^{\delta_\text{н}} &= c_{z\,\text{в.о}}^{\beta} n_\text{н} k_\text{в.о} k_{S\,\text{в.о}} \bar{y}_\text{в.о}, \\ m_y^{\delta_\text{н}} &= c_{z\,\text{в.о}}^{\beta} n_\text{н} k_\text{в.о} k_{S\,\text{в.о}} \bar{L}_\text{в.о}, \end{aligned}\right\} \quad (10.16)$$

where the $n_\text{н}=\frac{c_{z\,\text{в.о}}^{\delta}}{c_{z\,\text{в.о}}^{\beta}}$ - the coefficient of the relative rudder-effectiveness derivative; $n_\text{н}\approx\sqrt{\frac{S_\text{н}}{S_\text{в.о}}}$ with subsonic and $n_\text{н}\approx\frac{S_\text{н}}{S_\text{в.о}}$ at supersonic flight speeds;

$$\bar{y}_\text{в.о}=\frac{y_\text{в.о}}{l};\ \bar{L}_\text{в.о}=\frac{L_\text{в.о}}{l}.$$

compressibility effect on the rudder-effectiveness derivative caused by changes in the $c_{z\,\text{в.о}}^{\beta}$ and $n_\text{н}$ in M. The dependences of $m_x^{\delta_\text{н}}, m_y^{\delta_\text{н}}$ on M are similar $m_z^{\delta}=f(M)$ (see Fig. 9.15).

Torque/moment, created by ailerons. Ailerons are the control surfaces, suspended at wing tips and which are deflected to different sides. According to the taken rule of signs - positive is counted the aileron deflection of the right aileron down, and left - upward (Fig. 10.13). In flight during wing with the deflected down aileron, appears the supplementary positive normal force of $\Delta Y_э$, and at wing with the built up aileron - approximately the same in force intensity of opposite direction. Force couple of these creates the governing moment of roll of $M_{xэ}$. It is simultaneous, as a result of a change in the tangential components of the total aerodynamic force of wings, appears the yawing moment, directed to the side of wing with the omitted aileron (this barely is when using interceptors). However, during low angles of attack and small wing aspect ratio, the yawing moment, created by ailerons, is short and it can be disregarded.

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If we consider c_y^a constant on the wingspan, then an increase in the normal force it is possible to determine by formula

$$\Delta Y_{э} = c_y^{\alpha} n_{э} S'_{э} q \delta_{э},$$

a the coefficient of aileron-effectiveness derivative from formula

$$m_{x\,э}^{\delta} = -c_y^{\alpha} n_{э} k'_{S\,э} \bar{l}_{э}, \qquad (10.17)$$

where the $\bar{l}_{э} = \frac{l_{э}}{l}$ are a relative arm of the couple of the of $\Delta Y_{э}$, the approximately equal to the distance between the middles of aileron span, referred to the spread/scope of wing (Fig. 10.13); $k'_{Sэ} = \frac{S'_{э}}{S}$ are the relative wing area, occupied with aileron; $n_{э} = \frac{c_y^{\delta_{э}}}{c_y^{\alpha}}$ - the coefficient of relative aileron-effectiveness derivative; $n_{э} = \sqrt{\frac{S_{э}}{S'_{э}}}$ - with subsonic $(M < M_{кр})$ and $n_{э} \approx \frac{S_{э}}{S'_{э}}$ at supersonic flight speeds.

A change in the c_y^{α} in spread/scope is considered by

introduction to formula (10.17) of correction factor. At high angles of attack, the aileron-effectiveness derivative substantially descends, especially on aircraft with swept wings, which is connected with boundary-layer separation at the ends of the wings. To an increase in the aileron-effectiveness derivative contribute the used on such wings fences, slot or "teeth", the setting up of more carrying airfoil/profiles at wing tips, end slats.

10.5. Coefficients of the lateral static moments of aircraft.

In view of insufficient precise account by the calculated methods of the interference of aircraft components the moments of roll and yawing, which act on aircraft, is most reliable determined experimentally. Only at the stage of sketch design are applied the methods of the approximate determination of the lateral torque/moments.

The yawing moment is defined as sum of the aerodynamic couple of the M_y of the aircraft with neutral control, determined from formula (10.6), the torque/moment of the thrust/rod of the engine of M_{yp}, the torque/moment of $M_{y\delta H}$, created by rudder (10.16). Thus:

$$M_{y\,\text{с}} = M_y + M_{yp} + M_{y\delta\,\text{н}}$$

or, if we disregard the yawing moment from the engine thrust and to pass to moment coefficients, then we will obtain

$$m_{y\,\text{с}} = -c_z^{\beta}(\bar{x}_{\text{т}} - \bar{x}_{F\beta})\beta + c^{\beta}_{z\,\text{в.о}} n_{\text{н}} k_{\text{в.о}} k_{S\,\text{в.о}} \bar{L}_{\text{в.о}} \delta_{\text{н}}.$$

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Since β and δ_H they enter in this expression linearly, this same dependence let us present in the form

$$m_{y\,c}=m_y^{\beta}\beta+m_y^{\delta_н}\delta_н,$$

where the $m_y^{\beta}=-c_z^{\beta}(\bar{x}_т-\bar{x}_{F\beta})$; $m_y^{\delta_н}$ it is determined from expression (10.16).

The moment of roll, which appears with the slip of aircraft, we find as sum of the moments of the forces, which act on wing, tail assembly, and the torque/moment of wing-root interference effect, using dependences (10.11) - (10.13), (10.16) and (10.17). This will lead to result

$$M_{x\,c}=M_{x\,кр}+M_{x\,инт}+M_{x\,в.о}+M_{x\,н}+M_{x\,э},$$

or, passing over to moment coefficients:

$$m_{x\,\text{с}} = m_{x\,\text{кр}} + m_{x\,\text{инт}} + m_{x\,\text{в.о}} + m_{x\,\text{н}} + m_{x\,\text{э}} = (m^{\beta}_{x0} + m^{\beta}_{x\chi} + m^{\beta}_{x\psi})\beta + m^{\beta}_{x\,\text{инт}}\beta + m^{\beta}_{x\,\text{в.о}}\beta + m_x^{\delta_\text{н}}\delta_\text{н} + m_x^{\delta_\text{э}}\delta_\text{э}.$$

uniting the terms, which contain β, we will obtain

$$m_{x\,\text{с}} = m_x^{\beta}\beta + m_x^{\delta_\text{н}}\delta_\text{н} + m_x^{\delta_\text{э}}\delta_\text{э}, \qquad (10.19)$$

where

$$m_x^\beta = m_{x0}^\beta + m_{x\chi}^\beta + m_{x\psi}^\beta + m_{x\,\text{в.о.}}^\beta.$$

With the aid of relationships (10.18) and (10.19) it can be executed the analysis of lateral stability and aircraft handling.

Questions for repetition

1. Give the determination of the longitudinal and lateral foci of aircraft. Do coincide they between themselves?

2. Which two angular parameters in wing construction do make it possible substantially to change the rolling moment of aircraft?

3. Why with the slip of the axisymmetric isolated/insulated airplane fuselage does not appear rolling moment?

4. Why aerodynamic lateral forces and torque/moments do depend on slip angle and do not depend on roll attitude?

5. As they do affect direct/straight and sweepforward the lateral wing moments?

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Chapter XI. Forces and the ~~torque~~/moments, which act on aircraft during unsteady motion.

During the motion of aircraft along curved path to it, act the supplementary torque/moments, caused it by rotation/revolution. At low angles of attack, these torque/moments impede rotation/revolution; by their nature they are antitorque moments

either as them it is accepted to call, that hinder, or damping.

At near- or angles of attack beyond stalling, the torque/moment, caused by rotation/revolution, can invert of action, also, from that which damp shape rotating. In this chapter we will be restricted to the examination of the range of low angles of attack.

Besides the damping moments, on aircraft act the cross aerodynamic couples, also caused by rotation/revolution. Thus, for instance, rotation/revolution relative to axle/axis Oy_1 is the reason for the appearance not only of the damping yawing moment, but also the moment of roll; the rotation/revolution of relatively longitudinal axis Ox_1 produces the supplementary yawing moment. These cross aerodynamic couples, caused by the rotation/revolution of relatively orthogonal axle/axes, are called of spiral torque/moments.

In the examination of the torque/moments, caused by rotation/revolution, let us assume that is valid the stability hypothesis, according to which the instantaneous aerodynamic forces, applied to aircraft components, correspond to the instantaneous values of the angles of attack and slip, i.e., we will not accept

into consideration that circumstance that for producing the establish/installed value of aerodynamic force during a sudden change in the angular velocity is necessary finite time. Therefore the torque/moments in question are called stationary, or quasi-static.

The damping moments are created by jet power plant, which is connected with the action of Coriolis forces on the driving masses of gas through the circuit of engine during the rotation/revolution of aircraft. However, these torque/moments are short in comparison with aerodynamic and one should consider them during motion at the very low speeds or in the strongly rarefied atmosphere when aerodynamic couples are virtually equal to zero.

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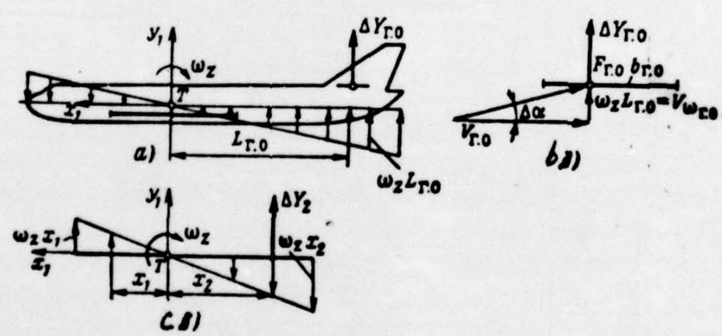

Fig. 11.1. Longitudinal damping moment: a) the diagram of the rotation/revolution of aircraft; b) a change in the angle of attack

of horizontal tail assembly; c) a change in the local angles of attack of wing.

Fig. 11.1. Longitudinal damping moment.

The major aircraft components, which create the longitudinal damping moment, are the horizontal tail assembly, the wing and partially fuselage (body). The longitudinal damping moment of aircraft as a whole is the sum of these torque/moments.

Torque/moment, created by horizontal tail assembly. If aircraft rotates relative to transverse axis Oz_1 at the angular velocity of ggggg, all points of aircraft, eliminated from rotational axis, they acquire peripheral speed (Fig. 11.1a).

Being restricted to the low values of angular velocity, it is possible to disregard a change in the module/modulus of the velocity vector of this point of aircraft as compared with flight speed. However, one should consider a change in the sense of the vector of speed, since the aerodynamic forces, which act on the section of aircraft in question, they depend on a change in the local angle of

attack.

If the average value of the peripheral speed in the field of the horizontal tail assembly is equal of $L_{г.о}\omega_z$, where the $L_{г.о}$ - the arm of horizontal tail assembly, then the angle of attack of horizontal tail assembly on the average will be changed by value (Fig. 11.1b)

$$\Delta\alpha=\frac{L_{г.о}\omega_z}{V_{г.о}}.$$

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If $\omega_z>0$, then angle of attack increases by $\Delta\alpha$. that will lead to an increase in the horizontal tail lift to value

$$\Delta Y_{г.о}=c^{\alpha}_{y\,г.о}q_{г.о}S_{г.о}\Delta\alpha.$$

relative to the center of mass this force it will give torque/moment

$$\Delta M_{z\,\text{г.о}} = -L_{\text{г.о}} \Delta Y_{\text{г.о}},$$

directed against the rotation/revolution of aircraft.

The coefficient of the damping moment, created by horizontal tail assembly, is equal to:

$$m_{z\,\text{г.о}} = -c^{\alpha}_{y\,\text{г.о}} \frac{S_{\text{г.о}} L^2_{\text{г.о}}}{S b_A^2} \sqrt{k_{\text{г.о}}}\, \bar{\omega}_z, \qquad (11.1)$$

where the $\bar{\omega}_z = \frac{\omega_z b_A}{V}$ is equal dimensionless angular velocity.

For the aircraft of the diagram of "weft" in formula (11.1) of $k_{\text{г.о}} = 1$.

When making these assumptions the dependence of $m_{z\,\text{г.о}}$ on dimensionless angular velocity is linear; therefore

$$m_{z_{\text{г.о}}}^{\bar{\omega}_z} = -c_{y\,\text{г.о}}^{\alpha} \frac{S_{\text{г.о}} L_{\text{г.о}}^2}{S b_A^2} \sqrt{k_{\text{г.о}}}. \qquad (11.2)$$

the wing moment. The portion/fraction of wing in producing the damping moment in usual airplane design with the rear arrangement of

tail assembly is low, which is explained by the arrangement/permutation of wing about the center of mass. However, this portion/fraction considerably grow/rises with the layout of aircraft by canard configuration or "bobtailed aircraft", and also during the application/use of swept or deltas.

If the wing moves at forward velocity of V and simultaneously rotates relative to the center of mass with angular velocity ω_z (see Fig. 11.1c), then at point with coordinate x_1, arrange/located before the center of mass, the supplementary velocity of motion, caused by rotation/revolution, is equal to $\omega_z x_1$, and at point with coordinate x_2, it comprises $\omega_z \dot{x}_2$. This distribution of normal velocities leads to a decrease in the local angle of attack at point with coordinate x_1 by $\Delta\alpha_1 \approx -\frac{\omega_z x_1}{V}$ and to an increase in the local angle of attack at point with coordinate x_2 by $\Delta\alpha_2 \approx -\frac{\omega_z x_2}{V}$, which will cause the appearance of negative increase of normal force in field x_1 and positively - in field x_2. Torque/moment from these forces is directed against rotation/revolution and depends on the angular velocity of ω_z.

In accordance with the hypothesis of bending, the examined picture is equivalent to bending of the center line of wing section

according to the law of $-\frac{\omega_z x}{V}$. By accepting this hypothesis, it is possible to calculate pressure distribution chordwise and to determine the damping moment theoretically.

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After manufacturing this bent model, by means of usual static blasting it is possible to determine the damping moment of wing and aircraft as a whole. Determined in this way the damping moment will be quasi-static.

Another way of experimental determination of the damping moment - this is the blasting of the undistorted model airplane, which accomplishes the induced or free oscillation/vibrations relative to axle/axis Oz_1. In this case is determined the damping moment taking into account unsteady flow condition.

The coefficient of the damping moment of wing will be is directly proportional to angular rate of rotation

$$m_{z\,\text{кр}} = m_{z\,\text{кр}}^{\bar{\omega}_z} \bar{\omega}_z,$$

where the derived $m_{z\,\text{кр}}^{\bar{\omega}_z}$ it can be obtained by the calculated or experimental way [1].

FOOTNOTE [1]. I. V. Ostoslavskiy, I. V. Strazheva. Flight dynamics. Stability and the controllability of flight vehicles. M., "machine-building", 1965. ENDFOOTNOTE.

The damping moment of fuselage is significantly less damping moment of wing and by the fact more tail assembly, and therefore at subsonic speeds it is determined in portion/fractions of the damping moment of the wing:

$$m_{z\phi}^{\bar{\omega}_z} \approx k_\phi m_{z\text{кр}}^{\bar{\omega}_z}, \qquad (11.3)$$

where the $k_\Phi = 0{,}15 \div 0{,}25$.

With M > 1, torque/moment of fuselage, caused by rotation/revolution, in certain cases can even change sign and shape rotating. For these Mach numbers, the coefficient of $m_{z\Phi}^{\bar{\omega}_z}$ can be determined by the approximation formula

$$m_{z\Phi}^{\bar{\omega}_z} = -2\frac{S_\Phi l_\Phi^2}{S b_A^2}\left[(\bar{l}_н - \bar{x}_\Phi)^2 - \frac{M_{об}}{l_\Phi^2 S_\Phi}\right],$$

where the S_Φ - area of maximum cross section of fuselage; l_Φ - the length of fuselage; $\bar{l}_н = \frac{l_н}{l_\Phi}$, $\bar{x}_\Phi = \frac{x_т}{l_\Phi}$ - the respectively relative length of the forward fuselage and the coordinate of the center of mass of aircraft, counted off from the leading edge/nose of fuselage;

$M_{об}$ - the torque/moment of the volume of fuselage relative to the center of mass of aircraft.

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[redacted] §11.2. Torque/moment, caused by the delay of downwash of tail assembly.

During the determination of the damping moment of the horizontal tail assembly, arrange/located behind the wing, was not considered from the circumstance that during a change in the angle of attack of $\left(\dot{\alpha}=\frac{d\alpha}{dt}\neq 0\right)$ a change in the downwash angle affects the lift of tail assembly not immediately, but with certain delay for a period of $t_з$, equal to the time of the displacement/movement of aircraft on the length of $L_{г.о}$.

The delay time of $t_з$ approximately will be equally to:

$$t_з \approx \frac{L_{г.о}}{V_{г.о}}.$$

consequently, the instantaneous value of rake angle in the field of horizontal tail assembly differs from the taken in the calculation torque/moment of tail assembly for value

$$\Delta\varepsilon = \frac{d\varepsilon}{d\alpha}\frac{d\alpha}{dt}t_з = \varepsilon^{\alpha}\frac{L_{г.о}}{V_{г.о}}\dot{\alpha}.$$

the corresponding increase of the pitching moment of horizontal tail assembly is equal to:

$$M_{z\varepsilon} = M_{z\varepsilon}^{\varepsilon_{\text{г.о}}} \Delta\varepsilon = M_{z\varepsilon}^{\varepsilon_{\text{г.о}}} \frac{L_{\text{г.о}}}{V_{\text{г.о}}} \varepsilon^{\alpha} \dot{\alpha},$$

or in dimensionless form

$$m_{z\varepsilon} = m_{z\varepsilon}^{\varepsilon_{\text{г.о}}} \frac{L_{\text{г.о}}}{V_{\text{г.о}}} \varepsilon^{\alpha} \dot{\alpha} = m_{z\,\text{г.о}}^{\bar{\omega}_z} \varepsilon^{\alpha} \frac{b_A}{V} \dot{\alpha}. \qquad (11.4)$$

if we introduce the concept of dimensionless derivative α by analogy with the dimensionless angular velocity:

$$\bar{\dot{\alpha}} = \dot{\alpha} \frac{b_A}{V}, \qquad (11.5)$$

that of formula (11.4) we will obtain

$$m_z^{\bar{\dot\alpha}} = \frac{\partial m_z}{\partial \dot\alpha}\,\frac{\partial \dot\alpha}{\partial \bar{\dot\alpha}} = m_{z\,\text{г.о}}^{\bar\omega_z}\varepsilon^{\alpha}. \qquad (11.6)$$

time lag of a change in the downwash it affects also the moment of wing and fuselage, but this effect it will be significant for the wing, established/installed after horizontal tail assembly, i.e., in the diagram of "weft". In other cases it can be disregarded.

The derivatives of $m_z^{\bar\omega_z}$ and $m_z^{\bar{\dot\alpha}}$ depend on Mach number (Fig.

11.2). A change in the $m_z^{\bar{\omega}_z} = f(M)$ is caused by the dependence of the lift effectiveness of aircraft components of M, while a change in the gggggg, furthermore, by the dependence of $m_z^{\bar{\omega}}$, on M.

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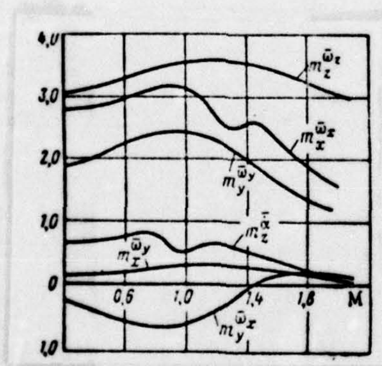

Fig. 11.2. Dependence of rotary and spiral derivatives of Mach number.

[redacted]§11.3. Forces and the torque/moments, caused by the rotation/revolution of aircraft relative to the longitudinal axis.

Torque/moments, created by wing. During the rotation/revolution of aircraft at the angular velocity of ggggg relative to longitudinal axis Ox_1 in cross sections of the being omitted wing, arranged/located at a distance z from axle/axis Ox_1 (Fig. 11.3, cross section B-B), the local angles of attack increase, and in the cross sections of the heaving wing (cross section A-A) they decrease by value

$$\Delta\alpha \approx \frac{z\omega_x}{V}.$$

this change in the angles of attack it leads to a change both of

the normal and in the tangential components total aerodynamic force of the cross section:

$$\left.\begin{aligned} dY_1 &\approx dY = c_y^{\alpha} qb\,\frac{\omega_x z}{V}\,dz, \\ dX_1 &\approx c_{x1}^{\alpha} qb\,\frac{\omega_x z}{V}\,dz, \end{aligned}\right\} \tag{11.7}$$

where

$$c_{x1}^{\alpha} \approx (c_x - c_y\alpha)^{\alpha} = c_x^{\alpha} - 2c_y - c_y^{\alpha}\alpha_0,$$

α_0 - zero-lift angle.

If we disregard the term of $c_y^{\alpha}\alpha_0$, to accept $c_x = c_{x0} + \frac{1+\delta}{\pi\lambda_0} c_y^2$ and to place $\frac{1+\delta}{\pi\lambda_0} = A$, then we will obtain

$$c_{x1}^{\alpha} \approx 2c_y(Ac_y^{\alpha} - 1).$$

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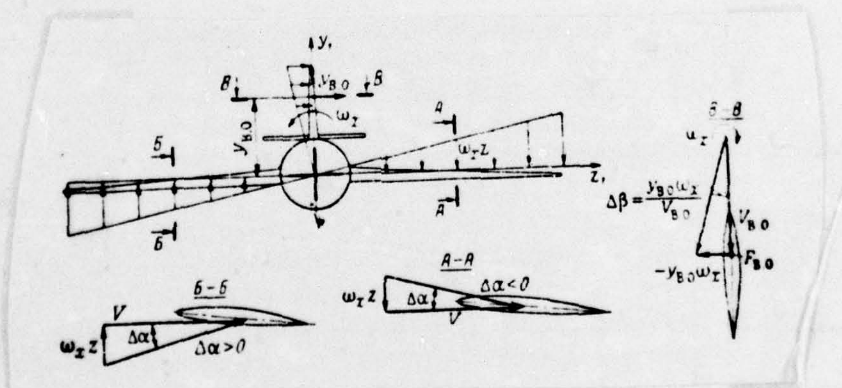

Fig. 11.3. Diagram of a change in the local angles of attack of wing and angle of vertical tail assembly during rotation/revolution relative to axle/axis Ox_1.

The redistribution of normal forces on spread/scope is the reason for the appearance of the moment of roll, while the redistribution of forces of periphery - the yawing moment. After accepting c_y^α as constant on the wingspan with that which was assigned c_y and by utilizing expression (11.7), we will obtain the approximate values moment derivative coefficient in the dimensionless angular velocity of $\bar{\omega}_x = \frac{\omega_x l}{2V}$ in the form

$$\left.\begin{aligned} m_{x\,\text{кр}}^{\bar{\omega}_x} &= -I_1 c_y^\alpha, \\ m_{y\,\text{кр}}^{\bar{\omega}_x} &= -2I_1 c_y (A c_y^\alpha - 1), \end{aligned}\right\} \quad (11.8)$$

where

$$I_1 = \frac{\lambda}{4} \int_{-1}^{1} \frac{b}{l} \bar{z}^2 d\bar{z}; \quad \bar{z} = \frac{2z}{l}.$$

for the tapered wing of $J_1 = \frac{1}{12\lambda} \frac{\eta+3}{\eta+1}$.

The comparison of derivatives (11.8) with derivatives, obtained by the calculation taking into account a change in the $C^{\alpha}_{y\,сеч}$, and also with the results of experimental studies makes it possible to introduce into formulas (11.8) correction factor k_1, equal to

$$k_1=0,9\left(0,5+0,0105\,\frac{1}{A}\right).$$

the compressibility effect of flow to the derivatives of $m_{x\,\text{кр}}^{\bar{\omega}_x}$ and $m_{y\,\text{кр}}^{\bar{\omega}_x}$ is caused by a change in the $c_y^{\alpha}=f(\mathrm{M})$ and $A=f(\mathrm{M})$. Since at the supersonic flight speeds of $A\approx\frac{1}{c_y^{\alpha}}$, value of the derivative of $m_{y\,\text{кр}}^{\bar{\omega}_x}$ is close to zero.

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The torque/moment, created by fuselage during the rotation/revolution of aircraft at the angular velocity of ω_x, is virtually equal to zero.

The torque/moments, created by vertical tail assembly during the rotation/revolution of aircraft relative to the longitudinal axis, are caused by a change in the angle of tail assembly (cross section B-B in Fig. 11.3).

An increase in the angle of vertical tail assembly in cross section is equal

$$\Delta\beta\approx\frac{\omega_x y_{в.о}}{V_{в.о}}.$$

the elementary lateral force, which acts in the cross section in question, is determined from formula

$$dZ_{в.о}=c^{\beta}_{z\,в.о}b_{в.о}qk_{в.о}\frac{\omega_x y_{в.о}}{V_{в.о}}dy_{в.о}.$$

this force relative to the center of mass of aircraft it creates the moments of roll and yawing which can be determined from equalities

$$\left.\begin{aligned} M_{x\,\text{в.о}} &= \int_0^{l_{\text{в.о}}} c_{z\,\text{в.о}}^{\beta} b_{\text{в.о}} q \sqrt{k_{\text{в.о}}} \frac{\omega_x y_{\text{в.о}}}{V} y_{\text{в.о}} dy_{\text{в.о}}, \\ M_{y\,\text{в.о}} &= \int_0^{l_{\text{в.о}}} c_{z\,\text{в.о}}^{\beta} b_{\text{в.о}} q \sqrt{k_{\text{в.о}}} \frac{\omega_x y_{\text{в.о}}}{V} L_{\text{в.о}} dy_{\text{в.о}}. \end{aligned}\right\} \quad (11.9)$$

hence with constant by height of $C^{\beta}_{z_{B.O}}$ we will obtain the following values moment derivative coefficient in the dimensionless angular velocity of the $\bar{\omega}_x$:

$$\left.\begin{aligned} m_{x\,\text{в.о}}^{\bar{\omega}_x} &= 2\sqrt{k_{\text{в.о}}}\,c_{z\,\text{в.о}}^{\beta}\,I_{\text{в.о}}, \\ m_{y\,\text{в.о}}^{\bar{\omega}_x} &= 2\sqrt{k_{\text{в.о}}}\,c_{z\,\text{в.о}}^{\beta}\,I'_{\text{в.о}}, \end{aligned}\right\} \qquad (11.10)$$

where

$$I_{\text{в.о}} = \int_0^{l_{\text{в.о}}} \frac{b_{\text{в.о}}\,y_{\text{в.о}}^2\,dy_{\text{в.о}}}{Sl^2}; \quad I'_{\text{в.о}} = L_{\text{в.о}} \int_0^{l_{\text{в.о}}} \frac{b_{\text{в.о}}\,y_{\text{в.о}}\,dy_{\text{в.о}}}{Sl^2};$$

$l_{\text{в.о}}$ - the height/altitude of vertical tail assembly; $b_{\text{в.о}}$ are the instantaneous value of chord.

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Since the $c^{\beta}_{\text{в.о}}<0,$ that $m^{\bar{\omega}_x}_{x\,\text{в.о}}$ and $m^{\bar{\omega}_x}_{y\,\text{в.о}}$ negative, i.e., during the rotation/revolution of the aircraft of relatively longitudinal axis vertical tail assembly creates the damping moment with respect to this axle/axis and the yawing moment, which attempts to turn the nose of aircraft to the side of its rotation/revolution.

For an aircraft as a whole, the transverse which damps (M_x) and the spirally turning up (M_y) torque/moments approximately are determined in the form of the sums of the torque/moments, created by wing and the vertical tail assembly:

$$\left.\begin{aligned} M_x &= M_x^{\bar{\omega}_x}\bar{\omega}_x = \left(m^{\bar{\omega}_x}_{x\,\text{кр}} + m^{\bar{\omega}_x}_{x\,\text{в.о}}\right) qSl\bar{\omega}_x, \\ M_y &= M_y^{\bar{\omega}_x}\bar{\omega}_x = \left(m^{\bar{\omega}_x}_{y\,\text{кр}} + m^{\bar{\omega}_x}_{y\,\text{в.о}}\right) qSl\bar{\omega}_x. \end{aligned}\right\} \quad (11.11)$$

it is necessary to keep in mind that the spiral torque/moments, created as wing and vertical tail assembly, on the subcritical Mach numbers have opposite signs; therefore total torque/moment in certain cases turns out to be so low that it they disregard. At supersonic rates of change in the angle of attack as a result of the rotation/revolution of aircraft does not lead to a change in the force of periphery, and the spiral yawing moment of wing is virtually equal to zero, i.e., in this case the spiral torque/moment of aircraft is determined by the spiral torque/moment of vertical tail assembly, in this case the yawing moment fosters the development of slip for the driving upward wing, which lowers damping the wing which

and without this decreases with an increase of Mach number. Therefore the spiral yawing moment one should consider during the calculation of the disturbed motion at supersonic speeds. The dependences of $m_x^{\bar{\omega}_x}=f(M)$ and $m_y^{\bar{\omega}_x}=f(M)$ are given in Fig. 11.2.

The rolling moment of the wing of $M_{x\,\text{кр}}$ is damping until the local angles of attack exceed critical value. The excess of critical angle of attack will lead not to an increase in the normal force to the being omitted wing, but to its decrease. Therefore $M_{x\,\text{кр}}$ can become equal to zero and even change sign to opposite, i.e., from that which damp shape rotating. At this angle of attack, is observed the autorotation - autogyration of aircraft. In more detail this question is examined by chapter XVI.

§11.4. Forces and the torque/moments, caused by the rotation/revolution of aircraft relative to axle/axis Oy_1.

During the rotation/revolution of aircraft at the angular velocity of the ω_y of the wing section and the fuselage, eliminated from axle/axis Oy_1 respectively up to distances z and x, they acquire the peripheral speed, equal $\omega_y z$ and $\omega_y x$ (Fig.

11.4). A change in the forces of periphery in the spread/scope of wing and transverse forces in fuselage and vertical tail assembly will be the reason for the appearance of the yawing moments and bank.

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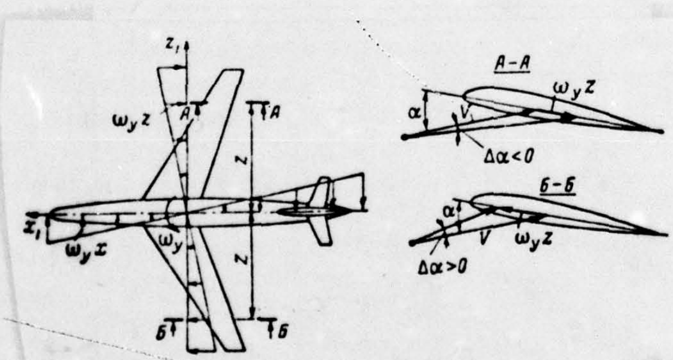

Fig. 11.4. Distribution of peripheral speed during the rotation/revolution of aircraft relative to normal axis.

Torque/moments, created by wing. The rotation/revolution of wing at the angular velocity of $\omega_y>0$ leads to a change in both velocity and the local angle of attack of cross section. With low α velocity increments and angle of attack in cross sections A-A the wing of aircraft are equal to:

$$\left.\begin{aligned}\Delta V &= \omega_y z \cos\alpha \approx \omega_y z,\\ \Delta\alpha &= -\frac{\omega_y z}{V}\sin\alpha = -\frac{\omega_y z}{V}\alpha,\end{aligned}\right\} \quad (11.12).$$

a in cross section B-B, symmetrical to cross section A-A, ΔV, $\Delta\alpha$ they will have opposite signs.

An increase in the tangential and normal forces, which act on the element of area of wing dS = bdz, we find from conditions

$$dX_1 \approx \frac{\partial X_1}{\partial V}\Delta V + \frac{\partial X_1}{\partial \alpha}\Delta\alpha,$$
$$dY_1 \approx \frac{\partial Y_1}{\partial V}\Delta V + \frac{\partial Y_1}{\partial \alpha}\Delta\alpha.$$

since $c_{x1} = f(M, \alpha), \quad c_{y1} = \varphi(M, \alpha)$ and with low α

$$c_{x1} \approx c_x - c_y\alpha,$$
$$c_{y1} \approx c_y,$$

after simple conversions obtain

$$\left.\begin{aligned} dX_1 &= \left[c_{x1}^{M}\mathrm{M} + 2c_{x1} - 2c_y\left(Ac_y^{\alpha} - 1\right)\alpha\right] qb\,\frac{\omega_y z}{V}\,dz, \\ dY_1 &= \left[c_y^{M}\mathrm{M} + c_y\right] qb\,\frac{\omega_y z}{V}\,dz. \end{aligned}\right\} \quad (11.13)$$

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These forces create moment with respect to axle/axes Ox_1 and Oy_1, equal respectively:

$$\left.\begin{aligned} dM_{y\,\text{кр}} &= -dX_1 z, \\ dM_{x\,\text{кр}} &= -dY_1 z. \end{aligned}\right\} \quad (11.14)$$

from expressions (11.13) and (11.14) by integration for the spread/scope of wing and subsequent differentiation with respect to the dimensionless angular velocity of $\overline{\omega}_y=\frac{\omega_y l}{2V}$ it is possible to determine derivatives of the yawing-moment coefficients of $M_{y\,\text{кр}}$ and bank of the $M_{x\,\text{кр}}$:

$$\left.\begin{aligned} m_{y\,\text{кр}}^{\bar{\omega}_y}&=-\left[c_{x1}^{\text{M}}\text{M}+2c_{x1}-2c_y\left(Ac_y^{\alpha}-1\right)\alpha\right]k_2 l_1,\\ m_{x\,\text{кр}}^{\bar{\omega}_y}&=-\left(c_y^{\text{M}}\text{M}+c_y\right)k_1 l_1,\end{aligned}\right\}\quad(11.15)$$

where I_1 is a dimensionless area moment of inertia of wing relative to axle/axis Ox_1 [see (11.8)]; k_1, k_2 are the correction factors, which consider the distribution of c_y^{α}, c_{x1}, c_{x1}^M, c_y^M according to spread/scope.

Torque/moment, created by fuselage. The spiral moment of roll of fuselage, characterized by the derivative of $m_{x\,\Phi}^{\bar{\omega}_y}$, is virtually equal to zero, and the damping yawing moment, characterized by the derivative of $m_{y\,\Phi}^{\bar{\omega}_y}$, is determined analogous with the longitudinal damping moment, i.e., for subsonic flight speeds

$$m_{y\,\Phi}^{\bar{\omega}_y}=k_{\Phi}\;m_{y\,\text{в.о}}^{\bar{\omega}_y},$$

where

$$k_{\Phi}=0{,}15\div 0{,}25.$$

for the supersonic flight speeds of $m_{y\,\text{ф}}^{\bar{\omega}_y}$ is determined from the aerodynamic theory of bodies of revolution.

Torque/moments, created by vertical tail assembly. The effective angle of vertical tail assembly, caused by the rotation/revolution of aircraft at the angular velocity of the ω_y, approximately it is possible to consider equal to

$$\Delta\beta \approx \frac{\omega_y L_{\text{в.о}}}{V_{\text{в.о}}}.$$

in this case to vertical tail assembly will act the lateral force

$$Z_{в.о}=c^{\beta}_{z\,в.о}S_{в.о}qk_{в.о}\frac{\omega_y L_{в.о}}{V_{в.о}},$$

which it creates moments with respect to axle/axes Ox_1 and Oy_1, equal respectively:

$$\left.\begin{aligned}M_{x\,в.о}&=Z_{в.о}y_{в.о},\\M_{y\,в.о}&=Z_{в.о}L_{в.о},\end{aligned}\right\} \qquad (11.16)$$

whence derivatives of the coefficients of these torque/moments in the dimensionless angular velocity of $\overline{\omega}_y$ they will take the form

$$\left.\begin{aligned} m_{x\,\text{в.о}}^{\bar{\omega}_y} &= 2c_{z\,\text{в.о}}^{\beta} k_{S\,\text{в.о}} \sqrt{k_{\text{в.о}}}\, \bar{L}_{\text{в.о}}\, \bar{y}_{\text{в.о}}, \\ m_{y\,\text{в.о}}^{\bar{\omega}_y} &= 2c_{z\,\text{в.о}}^{\beta} k_{S\,\text{в.о}} \sqrt{k_{\text{в.о}}}\, (\bar{L}_{\text{в.о}})^2, \end{aligned}\right\} \qquad (11.17)$$

where the $\bar{y}_{\text{в.о}}=\frac{y_{\text{в.о}}}{l}$ are a dimensionless coordinate of vertical tail assembly; $L_{\text{в.о}}=\frac{L_{\text{в.о}}}{l}$ - dimensionless vertical-tail length; $k_{S\,\text{в.о}}=\frac{S_{\text{в.о}}}{S}$ are a dimensionless coefficient.

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The torque/moments of aircraft as a whole, caused by rotation/revolution relative to axle/axis Oy_1, are determined from the formulas of the similarity:

$$\left.\begin{aligned} M_y &= m_y^{\bar{\omega}_y}\, qSl\,\bar{\omega}_y, \\ M_x &= m_x^{\bar{\omega}_y} qSl\,\bar{\omega}_y. \end{aligned}\right\} \qquad (11.18)$$

the first of the torque/moments is the torque/moment of directional damping of aircraft, and the second - by the spiral moment of roll of its.

The derivatives of $m_y^{\bar{\omega}_y}$ and $m_x^{\bar{\omega}_y}$ for an aircraft as a whole can be represented in the form of sums

$$\left.\begin{array}{l} m_y^{\bar{\omega}_y} = m_{y\,\text{кр}}^{\bar{\omega}_y} + m_{y\,\text{ф}}^{\bar{\omega}_y} + m_{y\,\text{в.о}}^{\bar{\omega}_y}, \\ m_x^{\bar{\omega}_y} = m_{x\,\text{кр}}^{\bar{\omega}_y} + m_{x\,\text{в.о}}^{\bar{\omega}_y}. \end{array}\right\} \qquad (11.19)$$

the torque/moment of directional damping, characterized to the derivative of $m_y^{\bar{\omega}_y}$, it increases on the subcritical Mach numbers and substantially descends with an increase of Mach number at supersonic flight speeds (see Fig. 11.2).

The spiral moment of roll acts in the direction of the driving back/ago wing. The dependence of $m_x^{\bar{\omega}_y} = f(M)$ is determined by a change in the lift effectiveness of wing and vertical tail assembly in Mach number.

During the determination of $m_y^{\bar{\omega}_y}$ and $m_x^{\bar{\omega}_y}$ was not considered the effect of the delay of the lateral downwash to the torque/moments, created by vertical tail assembly. However, this factor even in diagrams "high wing monoplane" and "low wing monoplane" does not exert a substantial influence on the value of moments with respect to axle/axes Ox_1 and Oy_1; therefore neglect it does not lead to appreciable error.

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PROBLEMS FOR REPETITION.

1. Which torque/moment from horizontal tail assembly does appear during the rotation/revolution of aircraft around axle/axis Oz_1 in angles of attack below stalling?

2. How does affect fuselage the damping moment with the return of aircraft relative to axle/axis Oz_1?

3. How does affect the angle of attack of horizontal tail assembly the delay of downwash?

4. By analyzing the dependence of $m_{z\,\text{г.о}}^{\bar{\omega}_z}$ on Mach number.

5. How does affect Mach number $m_{x\,\text{в.о}}^{\bar{\omega}_x}$?

6. How will be redistributed the angles of attack and speed of the flow about the half wings during the rotation/revolution of aircraft relative to axle/axis Oy_1 and which in this case do appear torque/moments - damping or rotating?

PROBLEM.

To determine the damping moment of the vertical tail assembly of aircraft an-24 in flight at height/altitude H = 2000 m at speed of V

= 400 km/h during rotation/revolution at the angular velocity of the $\omega_y = 0{,}175$ 1/sec. The fin-and-rudder area of $S_{в.о} = 13{,}4$ m², the coordinate of the center of pressure of vertical tail assembly on Ox_1 is equal to 11.32 m, the drag coefficient of the flow of $k_{в.о} \approx 1,\ c_{z\,в.о}^{\beta} = 2{,}22.$

Answer/response: $M_{y\,в.о}^{\omega_y} \omega_y \approx 37\,000$ nm.

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Chapter XII. Longitudinal static stability and aircraft handling.

Fig. 12.1. Longitudinal static stability of aircraft.

The concept about the static stability of aircraft as about inherent in it property to attempt without pilot's interference to retain the assigned flight conditions or to exhibit the tendency of return toward it, if for any reason appeared deviations from mode/conditions, is bygone determined in chapter VIII. Now, when are determined all the basic forces and the torque/moments, which act on aircraft, it is possible to give the quantitative evaluation of the static stability level of aircraft. This all the more important since knowledge of static stability level makes it possible also to evaluate the handlings of aircraft.

The flight conditions is characterized by the kinematic parameters α, β, γ, ϑ, V, H, etc. Usually longitudinal static stability examines relatively most important of the parameters of motion. Thus, for instance, for axial motion vital importance has static angle of attack stability (or c_y) and of flight speed, for yawing motion is important stability on roll attitude (lateral stability) and on angle (path or weathercock stability). On some of the parameters, the motions aircraft static stability do not possess (for example by height, to heading/course angle); however, this instability of significant effect on flight characteristics aircraft it does not exert.

Qualitative evaluation of the static stability of aircraft according to angle of attack can be made, by analyzing the relative location of the center of mass and longitudinal focus of aircraft. If with by direct-line flight aircraft hit the ascending gust of air, having speed W (Fig. 12.1), then angle of attack will obtain positive increase $\Delta\alpha$ that will cause in turn, the positive lift increment ΔY which is applied at the longitudinal focus of aircraft. The further behavior of aircraft will depend on the relative location of focus and center of mass.

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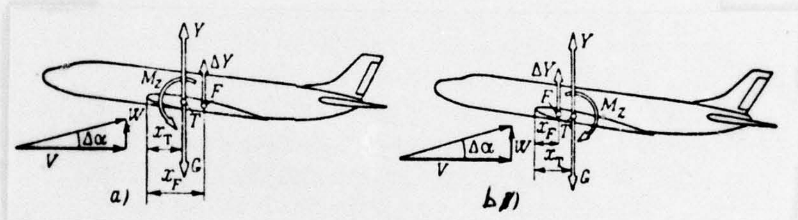

Fig. 12.1. Character of the effect of the relative location of the longitudinal focus and center of mass of aircraft on the stability of the equilibrium of the pitching moment: a) with the rear arrangement of focus; b) with the front/leading arrangement of focus.

If focus is arranged after the center of mass, $x_F > x_T$ (Fig. 12.1a), then unbalanced force ΔY creates the negative pitching moment of the $\Delta M_z < 0$, under action of which the aircraft will rotate until angle of attack decreases to the initial equilibrium value. During a random decrease in the angle of attack ($\Delta\alpha < 0$) of the direction of the action of lift increment ΔY and in the pitching moment, ΔM_z will be opposite those which were indicated in Fig. 12.1a, angle of attack will increase, strive for the initial mode/conditions.

With the front/leading arrangement of focus relative to the center of mass of $x_F < x_T$ (Fig. 12.1b) a random increase in the angle of attack produces the appearance of the pitching torque/moment of the $\Delta M_z > 0$, under action of which the angle of attack will even more increase, being driven out in value from equilibrium value. That means it is possible to draw the conclusion that the longitudinal static angle of attack stability is provided by the front/leading arrangement of the center of mass relative to the longitudinal focus of aircraft.

Quantitatively static stability can be evaluated, by analyzing the dependence of the coefficient of pitching moment on the lift coefficient with the constant Mach number. If, let us suppose this

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dependence is expressed by curve in Fig. 12.2 that pitching moments it will be balanced $(m_z=0)$ at three values of the c_y, which correspond to points A, B and C. During the flight conditions, which corresponds to point A, a random increase in the angle of attack increases c_y and produces the appearance of a negative pitching moment (point A'). In this case, the incremental value of the coefficient of pitching moment approximately is defined as

$$\Delta m_z = \frac{\partial m_z}{\partial c_y} \Delta c_y = m_z^{c_y} \Delta c_y.$$

in this case the particular derived $m_z^{c_y}$ is negative, i.e., the torque/moment of ΔM_z is reducing, whereupon the value of its coefficient is proportional to the value of the derivative of $m_z^{c_y}$.

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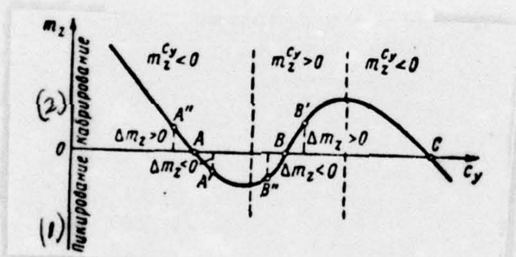

Fig. 12.2. Dependence of the coefficient of pitching moment on the coefficient of the longitudinal force with the constant Mach number.

Key: (1). Dive. (2). Pitching.

During a random decrease in the angle of attack (point A'') will appear pitching, i.e., that which reduce, torque/moment. The greater the module/modulus of the derivative of $|m_z^{c_y}|$, that the more righting moments. At point C, where also $m_z^{c_y}<0$, equilibrium as at point A, will be statically stable, that it is possible to show by similar considerations. Equilibrium at point B will be unstable, since at this point of $m_z^{c_y}>0$. Actually, in a random increase in the angle of attack of $(\Delta c_y>0)$ to new flight conditions it corresponds point B' and to aircraft it will act the pitching torque/moment of the $\Delta m_z>0$, which attempts to increase angle of attack.

Thus, the equilibrium of pitching moment is statically stable with $m_z^{c_y}<0$ and unstable with $m_z^{c_y}>0$; with $m_z^{c_y}=0$ the equilibrium will be indifferent (neutral); the derived $m_z^{c_y}$ is criterion, or degree, the longitudinal static stability.

From obtained previously dependence (9.54)

$$m_z^{c_y}=\bar{x}_T-\bar{x}_{Fc} \qquad (12.1)$$

follows that with $m_z^{c_y}<0$ must be made the inequality of $\bar{x}_{Fc}>\bar{x}_T$. That means longitudinal stability factor on c_y in

value is equal to the distance between centers of masses and the longitudinal focus of aircraft, divided into the length of the mean aerodynamic chord of aircraft.

The position of the longitudinal focus of aircraft is determined by dependence (9.53):

$$\bar{x}_F=\bar{x}_{F\,\text{кр}}+\left(\frac{1}{c_y^{\alpha}}-D\right)c^{\alpha}_{y\,\text{г.о}}\,k_{\text{г.о}}A_{\text{г.о}}+\sum\Delta\bar{x}_{Fi}. \quad (12.2)$$

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Since with $M>M_{\text{кр}}$ the center of pressure is displaced back/ago, with a further increase in Mach number is displaced

back/ago the focus of aircraft, which in the fixed position of the center of mass produces an increase in the static stability level in c_y.

In aviation practice the longitudinal stability factor of $m_z^{c_y}$ is standardized. For maneuverable aircraft, for example fighters, the stability level must be less than of nonmaneuverable - the transport and especially passenger aircraft for which the stability level must be not the less determined value for each aircraft type in order to have the determined "margin" of stability. Therefore in accordance with equality (12.1) a difference in the $\bar{x}_{Fc}-\bar{x}_T$ is called usually the reserve of centering.

The reserve of centering increases either by the displacement of the center of mass forward (by corresponding arrangement/permutation of the loads and passengers), or by the displacement of focus back/ago. Of the aircraft of usual diagram, the displacement of focus back/ago, in particular, can be reached by an increase in the coefficient of the area moment ratio of tail assembly. Since criterion for stability

$$m_z^{c_y} = \frac{\partial m_z}{\partial c_y}, \text{ и } n_y = \frac{c_y S \frac{\varrho V^2}{2} - G\cos\theta}{G},$$

at the fixed speed

$$m_z^{c_y} = m_z^{n_y} S \frac{\varrho V^2}{2G}.$$

Therefore stability criterion on c_y otherwise is called the criterion for static stability on g-force. Stable on g-force aircraft attempts to hold the rated value of g-force automatically.

During the analysis of longitudinal stability of g-force, was assumed to be the invariability of flight speed during the action of disturbance/perturbations on aircraft or independence of the force coefficients and torque/moments from speed and Mach number of flight. Experiment shows, however that to the different Mach numbers correspond the different dependences of $m_z = f(c_y)$ (Fig. 12.3).

We examine the effect of speed and, therefore, Mach number on an example of level flight. In this case the weight of aircraft is equal to lift and normal load factor

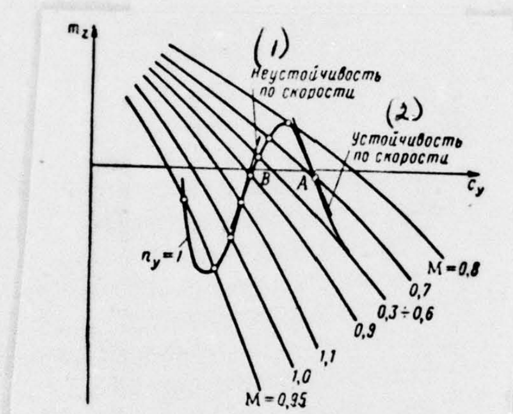
$$n_y = \frac{c_y S \varrho a^2 \mathrm{M}^2}{2G} = 1.$$

from the last/latter expression we have:

$$c_y = \frac{2G}{S\varrho a^2} \frac{1}{\mathrm{M}^2}. \qquad (12.2a)$$

Fig. 12.3. Effect of Mach number on the coefficient of pitching moment.

Key: (1). Speed instability. (2). Speed stability.



Hence for each value m we find the completely determined value c_y.

through the obtained value of c_y and the assigned Mach number in Fig. 12.3 we will obtain the determined value of m_z. If we apply the points, which correspond to the vapors M c_y, and to connect them, then is obtained curvedly (heavy line in figure), that corresponds to the normal load factor, equal to unity.

If flight conditions is changed so that the point, which corresponds to new mode/conditions, will lie/rest on curve for $n_y=1$, then will be changed not only c_y, but also Mach number of flight; therefore along this the curved coefficient of the pitching moment of m_z is the function of c_y and M. Under these conditions as stability criterion, it is more correct to take the partial, but complete derivative:

$$\frac{dm_z}{dc_y}=\frac{\partial m_z}{\partial c_y}+\frac{\partial m_z}{\partial M}\frac{dM}{dc_y}=m_z^{c_y}+m_z^{M}\frac{dM}{dc_y}. \quad (12.3)$$

differentiating expression (12.2a) with respect to c_y, we find

$$\frac{dM}{dc_y}=-\frac{M}{2c_y}.$$

after substituting the value of $\frac{dM}{dc_y}$ into equation (12.3), we will obtain

$$\frac{dm_z}{dc_y} = m_z^{c_y} - \frac{M}{2c_y} m_z^M. \tag{12.4}$$

since relationship (12.4) it considers a change of Mach number in level flight, it is called also the criterion for the longitudinal static speed stability of flight and characterizes the longitudinal static stability of aircraft in level flight with variable speed.

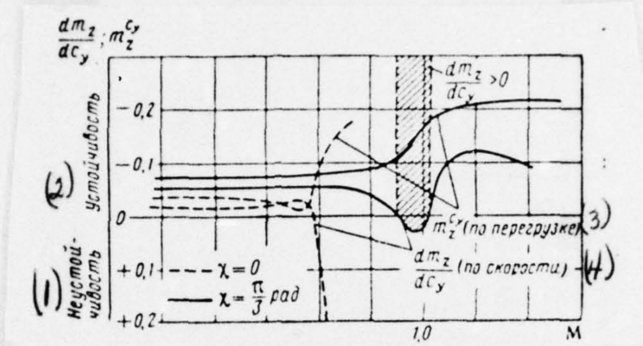


Fig. 12.4. Effect of Mach number on stability criterion on g-force and on speed.

Key: (1). Instability. (2). Stability. (3). (on g-force). (4). (on speed). (5). is glad.

At low flight speeds when $M<M_{кр}$, the moment characteristics of aircraft do not depend on Mach number of $(m_z^M=0)$, therefore from the comparison of expressions (12.1) and (12.3), it follows that for these values of Mach numbers the criteria for the longitudinal static stability on g-force and on speed are equal.

For the majority of contemporary aircraft, a further increase in Mach number leads to the appearance of the diving pitching moment as a result of the displacement of the focus of aircraft back/ago, i.e., with $M>M_{кр}$ the coefficient of $m_z^M<0$. In this case of terms in the right side of equality (12.4) different signs and, therefore, with the large Mach numbers

$$\left|\frac{dm_z}{dc_y}\right|<\left|\frac{\partial m_z}{\partial c_y}\right|,$$

i.e. static stability level on g-force are more static stability level on speed. In the transonic range of Mach numbers, the aircraft can "lose" speed stability, and, the more the sweepback of the wing of aircraft, the smoother change the stability criteria on g-force and on speed with an increase in Mach number (Fig. 12.4). The zone of

Mach numbers, within which the $\frac{dm_z}{dc_y} > 0$, is called the zone of time/temporary loss of stability on speed.

§12.2. The longitudinal balance of forces and torque/moments in rectilinear adjusted motion.

In the direct-line steady motion normal and forces of periphery and pitching moments must be balanced (balanced).

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In flight without bank and slip (axle/axes Oz and Oz_1 coincide) of the force condition of equilibrium and torque/moments it is possible to write in the form

$$P - X - G\sin\theta = 0, \quad Y - G\cos\theta = 0, \quad M_z = 0.$$

in axial motion the pilot control/guides the thrust of engine, the drag of aircraft (aerodynamic brake) and the horizontal tail lift, caused by the deflection of elevator. The force conditions of equilibrium, which act in fore-and-aft plane, are investigated in sufficient detail in chapter III and IV; therefore in the present chapter are analyzed the trimmed conditions of pitching moment under the already balanced forces. If pitching moment is balanced, then the coefficient of pitching moment, determined by relationship (9.51), will be equal to zero, i.e.,

$$m_z = m_{z0} + m_z^{c_y} c_y + m_z^{\varphi} \varphi + m_z^{\delta_{\text{н}}} \delta_{\text{н}} + m_{zp} = 0. \qquad (12.5)$$

Pitching moment can be balanced by the deflection of control of the angle of $\delta_{\text{н}}$, determined from condition (12.5):

$$\delta_{\text{н}} = -\frac{m_{z0} + m_z^{c_y} c_y + m_z^{\varphi} \varphi + m_{zp}}{m_z^{\delta_{\text{н}}}}, \qquad (12.6)$$

or taking into account formulas (9.51) and (9.52) we will obtain

$$\delta_{в}=-\frac{m_{z0}+(\bar{x}_{т}-\bar{x}_{F})c_{y}+m_{zp}}{c^{\alpha}_{y\,г.о}k_{г.о}A_{г.о}n_{в}}+\frac{\varphi}{n_{в}}. \qquad (12.7)$$

relationship (9.51), from which is obtained the trimmed conditions (12.5), it makes it possible to confirm that on the strength of the linear dependence of the $m_z=f(\delta_в)$ of the curve/graph of the function of $m_z=f(c_y)$ for the different surface positions with of low c_y and the $\delta_в$ will be equidistant curves (Fig. 12.5); equidistant will be and a curve/graph $m_z=f(\alpha)$.

The balancing elevator angle (with M = const) linearly depends on lift coefficient, and, if aircraft is statically stable $(m_z^{c_y}<0$, or, that the same, $\bar{x}_т<\bar{x}_F)$, then with an increase in the c_y the value of $\delta_в$ will decrease (Fig. 12.6a), since $m_z^{\delta_в}<0$ (9.52). For an unstable aircraft, on the contrary, in proportion to the increase in the c_y will increase the $\delta_в$. On

the basis of the taken rule of the signs of ($\delta_{\text{в}}>0$ during droop), the elevator of stable aircraft with the tail arrangement of tail assembly is deflected upward in proportion to the increase in the c_y (angle of attack).

At low flight speeds at high angles of attack as a result of boundary-layer separation, the linearity of $\delta=f(c_y)$, and other aerodynamic characteristics, is disturbed.

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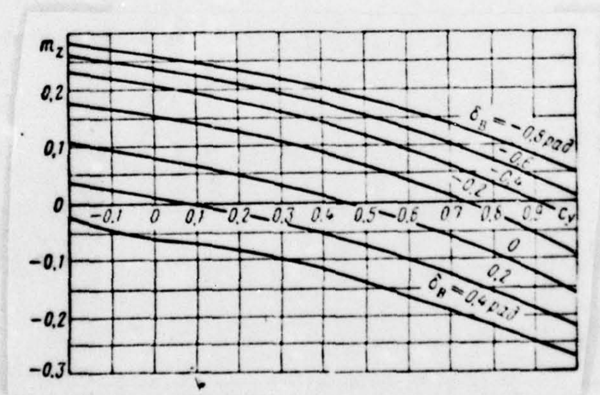

Fig. 12.6. Effect of the deflection of elevator of the pitching moment of aircraft.

рад = degree

The curve/graphs, given in Fig. 12.6a are called balancing curves on g-force.

By substituting the known value of $c_y=\frac{2G}{\rho SV^2}$ in expression (12.6), it is possible to obtain the direct coupling between the flight speed of V and the elevator angle of the δ_B, which no longer will be linear (Fig. 12.6b). The plotted function of $\delta_B=f(V)$ is called balancing curve on speed.

For the preservation/retention/maintaining of the value of lift in proportion to the increase in the velocity at level flight it is necessary to decrease the angle of attack, for which elevator one should gradually omit in order to create negative pitching moment. At the same time for the balance of torque/moment at the reduced angle of attack control also must be deflected down (Fig. 12.6), if aircraft is stable, and, on the contrary, upward, if aircraft is unstable. Thus, simultaneous balance at forces and torque/moments requires satisfaction of the following condition:

$$\frac{d\delta_{\text{в}}}{dc_y} < 0,$$

consequently:

$$\frac{d\delta_{\text{в}}}{dV} > 0,$$

that taking into account (12.6) it gives:

$$\frac{m_z^{c_y}}{m_z^{\delta_{\text{в}}}} < 0.$$

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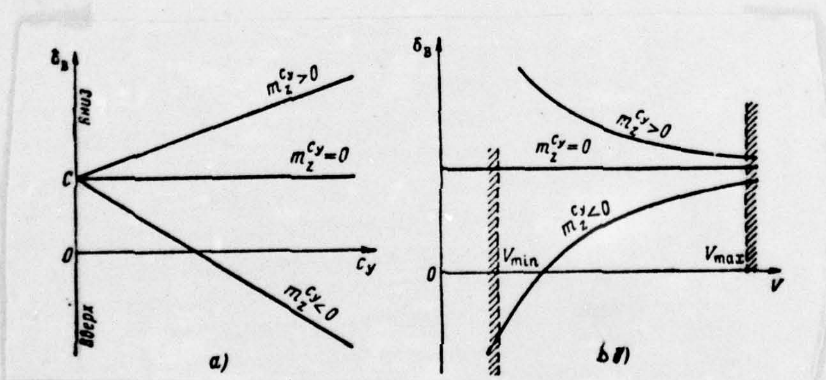


Fig. 12.6. Balancing curves of the subsonic aircraft: a) on g-force; b) on velocity.
Key: (1) upward. (2) down.

Since $m_z^{\delta_{\text{в}}}<0,$ condition of the absence of the double displacement/movement of control to one side for the balance of forces and into opposite for the balance of torque/moments, it is possible to express by inequality

$$m_z^{c_y}<0,$$

i.e. aircraft it must possess the longitudinal static stability on g-force.

From the stricter analysis which here is not given, it follows that taking into account compressibility for the absence of the double displacement/movement of control is necessary satisfaction of the following condition:

$$\frac{dm_z}{dc_y}<0.$$

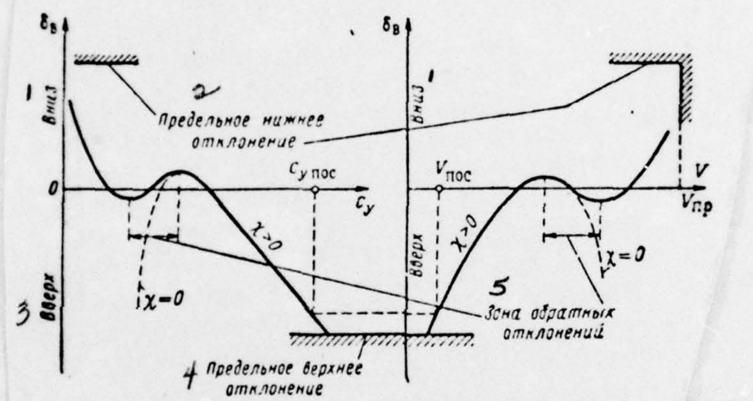
unfortunately, this condition, as follows from Fig. 12.1, it is disturbed in transonic speed range (Mach numbers). In this range changes the law governing the deflection of elevator depending on c_y or V (Fig. 12.7). With an increase in the velocity of flight for the balance of torque/moment, the elevator is necessary to deflect not down, but upward (reverse deflection of control).

In this case due to the center-of-pressure travel, back/ago appears the negative pitching moment, for balancing of which is required the positive displacement. The negative pitching moment, caused by the change of the center of pressure back/ago, predominates above the pitching torque/moment, caused by a decrease in the increase in the lift coefficient. This phenomenon especially strongly

is exhibited at straight-wings airplane at whose center of pressure and focus sharply are moved back/ago with the onset of shock stall.

Fig. 12.7. Balancing curves of the supersonic aircraft: a) on g-force; b) on velocity.

Key: (1). Down. (2). Maximum lower deviation. (3). Upward. (4). Maximum upper deviation. (5). Zone of reverse deviations.



A decrease in the downwash angle in proportion to the increase in Mach number of flight also contributes to the emergence of negative pitching moment due to an increase in the true angle of attack of horizontal tail assembly. The negative pitching moment, which appears with $M>M_{кр}$, is so great of straight-wings airplane, that in many instances limits a further increase in Mach number of flight, since for the balance of aircraft are required the large deflections of elevator (dotted line in Fig. 12.7). Of aircraft with swept and delta wing, the phenomenon "tightening into dive" is considerably weaker. Of these aircraft at transonic speed, the "tightening into dive" is not accompanied by the large deflections of elevator and by stick forces. It is continued to Mach number, close to unity, whereupon again appears the pitching torque/moment, for the balancing of which elevator must be deflected down. This section in balancing curves is conventionally designated as in practice "spoon".

Relationship (12.7) makes it possible to examine the effect of the position of the center of mass of aircraft (effect of centering) to the deflection of elevator. If in flight the center of mass was misaligned forward, after disturbing balance, then on aircraft acts the negative pitching moment, for countering of which elevator must be deflected upward. That means a decrease in the value of $\bar{x}_T$ requires a decrease in the angle of deflection of δ_B. From

equalities (12.6) and (12.7) it is possible to determine

$$\Delta\delta_{\text{в}} = \frac{-c_y}{c^{a}_{y\,\text{г.о}}\,k_{\text{г.о}}A_{\text{г.о}}n_{\text{в}}}\Delta\bar{x}_{\text{т}} = -\frac{c_y}{m_z^{\delta_{\text{в}}}}\Delta\bar{x}_{\text{т}}.$$

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Thus, the more the c_y, the more will be required the positive displacement. The most dangerous is the displacement of the center of mass forward under the conditions of the landing when are realized the quite large in flying practice values of c_y. Sometimes this displacement is so great that even the maximum deflection of elevator does not make it possible to balance the appearing pitching moment, but indeed it is always necessary to have certain reserve in the deflection of control (12-20o/o of the maximum) for the countering of disturbance/perturbations or maneuver accomplishment.

The maximum front/leading position of the center of mass (forward limit) is determined by the solution to equation (12.6) relative to $x_т$ in the absence of the thrust/rod of $(m_{zp}=0)$ and taking into account the fact that the $m_z^{c_y}=\bar{x}_т-\bar{x}_F$:

$$\bar{x}_{т\ пр.пер}=\bar{x}_{F\ пос}-\frac{m_{z0\ пос}+m_z^{\varphi}\varphi+m_z^{\delta}\delta_{пос}}{c_{y\ пос}}. \qquad (12.8)$$

all values, which enter in (12.8), are determined for the conditions of landing, i.e., for the landing configuration of aircraft and taking into account the proximity of the Earth. If steady-state stability factor on the g-force of $m_z^{c_y}$ is regulated, then maximally stern heaviness it is possible to find from condition

$$\bar{x}_{\text{т.пр.задн}} = \bar{x}_F - \left|m_z^{c_y}\right|. \qquad (12.9)$$

thus, at the standard conditions of flight static stability and controllability will be provided for, if are satisfied conditions

$$\bar{x}_{\text{т.пр.пер}} < \bar{x}_{\text{т}} < \bar{x}_{\text{т.пр.задн}} < \bar{x}_F.$$

the distance between maximum front/leading and stern heaviness it is called the operating range of centering.

With the realization of the longitudinal balance of aircraft by rotary stabilizer (but not by elevator) from equation (12.5), by set/assuming $\delta_{в}=0$, it is possible to obtain expression for determining the balancing angle of deflection of the stabilizer:

$$\varphi = -\frac{m_{z0} + m_z^{c_y} c_y + m_{zp}}{m_z^{\varphi}}. \qquad (12.10)$$

for the deflected stabilizers as and elevators, are constructed balancing curves on g-force and on speed. At present on large subsonic aircraft is applied the combination of the elevator and deflected stabilizer.

~~Fig.~~ §12.3. The hinge moment of height/altitude.

The resultant of the aerodynamic forces, which act on elevator, R_b are applied at a distance of d_b from hinge line (Fig. 12.8a).

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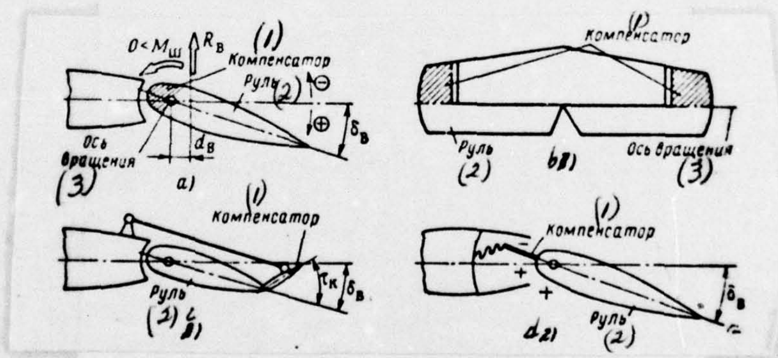

Fig. 12.8. Basic forms of aerodynamic balance: a) axial; b) horn; c) balance tab; d) internal.

Key: (1) Compensator. (2). Control. (3) Rotational axis.

Its torque/moment relative to hinge line is called the hinge moment:

$$M_{\text{ш}} = -R_{\text{в}} d_{\text{в}}. \qquad (12.11)$$

this determination of hinge moment visually, but inconveniently for practical calculations, since the value of the arm of the $d_{\text{в}}$ of the aerodynamic force of $R_{\text{в}}$ depends on the elevator angle of $\delta_{\text{в}}$, angle of attack of the horizontal tail assembly of $\alpha_{\text{г.о}}$ and Mach number of flight. The usually hinge moment is defined by the method of aerodynamic similarity as product of the hinge-moment coefficient of $m_{\text{ш}}$ to velocity head q, area of $S_{\text{в}}$ and the chord of the $b_{\text{в}}$ of elevator:

$$M_{\text{ш}} = m_{\text{ш}} q_{\text{г.о}} S_{\text{в}} b_{\text{в}} = m_{\text{ш}} k_{\text{г.о}} q S_{\text{в}} b_{\text{в}}. \qquad (12.12)$$

the hinge moment of $M_{\text{ш}}$ is positive, if it attempts to deflect elevator down.

For a decrease in the hinge moment, is applied aerodynamic balance whose target/purpose entails a decrease in the arm of the $d_{\text{в}}$ of the aerodynamic force of elevator. The most widely accepted forms the compensations are: axial, horn, internal, servo compensation (Fig. 12.8) and trim tabs. During overhang balance (Fig. 12.8a) the rotational axis of control is displaced to the side of center of pressure; during horn balance the remote forward part of the control displaces forward center of pressure (Fig. 12.8b). Servo compensation (Fig. 12.8c) it entails the automatic deflection of the servo-surface to the side, opposite to the deflection of control. In this case even small, but eliminated from the rotational axis of control aerodynamic force in the servo-surface makes it possible to decrease the hinge moment. During internal compensation the center-of-pressure travel forward is achieved by the use of a pressure difference in the cavity, divided by flexible partition/baffle (Fig. 12.8d).

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Trim tab differs from servo tab in terms of the fact that it is

control/guided by pilot, and his deflection of (τ_B) does not depend on surface position. Unlike the forms of compensation pointed out above the trim tab remove/takes forces only with one fixed flight conditions. Is most widely common overhang balance as sufficiently simple and effective.

Aerodynamic balance makes it possible to decrease the hinge moment, but to zero decreased it should not be, since pilot in this case will not "feel" control and can allow errors during a change in the flight conditions. In endurance flight in one mode/conditions the hinge moment can be decreased down to zero with the aid of trim tab.

From materials of tests, it follows that the elevator hinge-moment coefficient is the linear function of the deviation of control of altitude, of the angle of attack of horizontal tail assembly and angle of deflection of trim tab. Consequently, in the operational range of angular deflection of elevator hinge-moment coefficient can be determined by formula

$$m_{ш}=m_{ш}^{\delta_{в}}\delta_{в}+m_{ш}^{\alpha_{г.о}}\alpha_{г.о}+m_{ш}^{\tau_{в}}\tau_{в}, \quad (12.13)$$

where the $\tau_{в}$ - the angle of deflection of the elevator trim tab; $m_{ш}^{\delta_{в}}$, $m_{ш}^{\tau_{в}}$, $m_{ш}^{\alpha_{г.о}}$ are derivatives of hinge-moment coefficient in the elevator angles and trim tab, the angle of attack of horizontal tail assembly, i.e.,

$$m_{ш}^{\delta_{в}}=\frac{\partial m_{ш}}{\partial \delta_{в}}; \quad m_{ш}^{\alpha_{г.о}}=\frac{\partial m_{ш}}{\partial \alpha_{г.о}}; \quad m_{ш}^{\tau_{в}}=\frac{\partial m_{ш}}{\partial \tau_{в}}.$$

the derivatives of $m_{ш}^{\delta_{в}}$ and $m_{ш}^{\alpha_{г.о}}$ depend on type and size/dimensions of the balancing of control surfaces of height/altitude. For an elevator with overhang balance during the precomputations these derivatives can be determined by the following

empirical formulas:

$$m_{\text{ш}}^{\delta_{\text{в}}} = -0{,}14\left[1 - 6{,}5\left(\frac{S_{\text{о.к}}}{S_{\text{в}}}\right)^{3/2}\right]c_{y\,\text{г.о}}^{\alpha}; \qquad (12.14)$$

$$m_{\text{ш}}^{\alpha_{\text{г.о}}} = -0{,}12\,\frac{S_{\text{в}}}{S_{\text{г.о}}}\left(1 - 3{,}6\,\frac{S_{\text{о.к}}}{S_{\text{в}}}\right)c_{y\,\text{г.о}}^{\alpha}, \qquad (12.15)$$

where the $S_{\text{о.к}}$ - the area of overhang balance.

With an increase in the ratio of the area of overhang balance to the area of the elevator of $\frac{S_{\text{о.к}}}{S_{\text{в}}}$ the derivatives of $m_{\text{ш}}^{\delta_{\text{в}}}$, and , $m_{\text{ш}}^{\alpha_{\text{г.о}}}$. decrease and with $\frac{S_{\text{о.к}}}{S_{\text{в}}} = 0{,}28$ become zero. A further increase in the area of overhang balance leads to the overcompensation of elevator, which is inadmissible, since in this case pilot experience/tests opposite on sign stick force.

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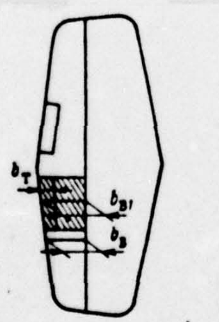
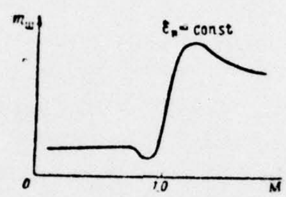


Fig. 12.9. Arrangement of trim tab on elevator.

Fig. 12.10. Effect of Mach number on hinge-moment coefficient.



The change in the hinge-moment coefficient, caused by the deflection of the elevator trim tab, is determined by the following empirical formula:

$$\Delta m_{ш}^{\tau_{в}} = -\frac{S_{т} b_{в1}}{S_{в} b_{в}} \sqrt{\frac{b_{в1}}{b_{т}}}\, \tau_{в}, \qquad (12.16)$$

where the $S_{т}$ - the area of trim tab; the $b_{в1}$ is determined the mean chord of that part of the control on which is arranged trim tab; the $b_{в}$ are the mean chord of entire elevator; $b_{т}$ - the chord of trim tab (Fig. 12.9); the $\tau_{в}$ - angle of deflection of trim tab in radians. The compressibility of air has a great effect on the value of hinge-moment coefficient.

With the onset of shock stall, the center of pressure of elevator is moved back/ago; therefore the hinge moment by transonic speeds sharply grow/rises. At transonic and supersonic speeds hinge moment is so great which is difficult to control/guide aircraft without hydraulic booster. The character of a change in the hinge-moment coefficient of the $m_{ш}$ of elevator in flight mach number is shown in Fig. 12.10.

For supersonic speeds the derivatives of $m_{ш}^{\delta_{в}}$ and $m_{ш}^{\delta_{г.о}}$ can be determined analytically, by utilizing theory of the supersonic similarity:

$$m_{ш}^{\delta_{г.о}}=m_{ш}^{\delta_{в}}=-\frac{2}{\sqrt{M^{2}-1}}\left(1-\frac{2S_{о.к}}{S_{в}}\right). \qquad (12.17)$$

12.4. Force feel of elevator. Stability of aircraft with the freed control.

Elevator connect with steering control, knob/stick or the control actuator of autopilot. For determining force feel from elevator, is utilized the virtual work principle, according to which the work of forces, equilibrous, is equal to zero during any virtual displacements. For the balancing of hinge moment to control lever (to steering control, to knob/stick, the stock/rod of the control actuator of autopilot) it is necessary to make the effort of $P_{\text{в}}$. Virtual displacements are the displacements of control lever along the longitudinal axis of the aircraft of $dx_{\text{в}}$, and deflection of the elevator of $d\delta_{\text{в}}$ (Fig. 12-11). Let us compose the equation of works during virtual displacements, by disregarding forces of friction in the control system:

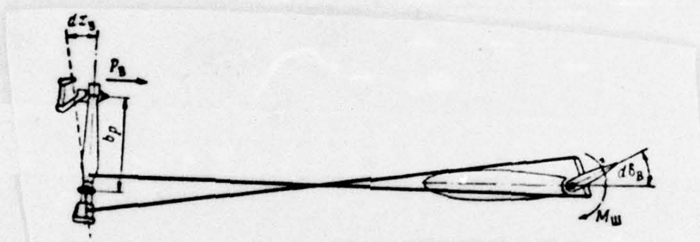


Fig. 12.11. Diagram of the transfer of force to the steering control of the elevator control.

$$P_{в}dx_{в}+M_{ш}d\delta_{в}=0.$$

Hence force feel of elevator

$$P_{в}=-k_{ш}M_{ш}, \qquad (12.18)$$

where the $k_{ш}=\frac{d\delta_{в}}{dx_{в}}$ are gear ratio, 1/m.

Positive it is considered pressing (from itself), and negative - the pulling (on itself) force. By substituting in equality (12.18) the value of hinge moment from relationship (12.12), taking into account (12.13) we will obtain

$$P_{в}=-k_{ш}k_{г.о}m_{ш}qS_{в}b_{в}. \qquad (12.19)$$

In turn,, hinge-moment coefficient it is determined by relationship (12.13); therefore force feel of elevator can be presented in the form:

$$P_{в}=-k_{ш}k_{г.о}(m_{ш}^{\delta_{в}}\delta_{в}+m_{ш}^{\alpha_{г.о}}\alpha_{г.о}+m_{ш}^{\tau_{в}}\tau_{в})\,qS_{в}b_{в}. \qquad (12.20)$$

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Is of interest the determination of the effect of the displacement of the center of mass of aircraft to pressure on

knob/stick. During the low displacement of the center of mass it is possible to write

$$\Delta P_{\text{в}} = P_{\text{в}}^{\bar{x}_{\text{т}}} \Delta \bar{x}_{\text{т}}. \qquad (12.21)$$

The particular derived $P_{\text{в}}^{\bar{x}_{\text{т}}}$ can be determined from relationship (12.20), in which the only elevator angle depends on position of center of gravity in aircraft:

$$P_{\text{в}}^{\bar{x}_{\text{т}}} = \frac{\partial P_{\text{в}}}{\partial \bar{x}_{\text{т}}} = \frac{\partial P_{\text{в}}}{\partial \delta_{\text{в}}} \frac{\partial \delta_{\text{в}}}{\partial \bar{x}_{\text{т}}} = -k_{\text{ш}} k_{\text{г.о}} q S_{\text{в}} b_{\text{в}} m_{\text{ш}}^{\delta_{\text{в}}} \delta_{\text{в}}^{\bar{x}_{\text{т}}}. \qquad (12.22)$$

In turn,, the particular derived $\delta_{\text{в}}^{\bar{x}_{\text{т}}}$ it is possible to determine from formula (12.6):

$$\frac{\partial \delta_{в}}{\partial \bar{x}_{т}} = \delta_{в}^{\bar{x}_{т}} = -\frac{c_y}{m_z^{\delta_{в}}}.$$

Substituting this value of $\delta_{в}^{\bar{x}_{т}}$ in equality (12.22) and taking into account that in level flight the lift is equal to gravitational force, i.e., $c_y = \frac{G}{qS}$, we will obtain expression for a change in the force feel of elevator during a change in the centering:

$$P_{в}^{\bar{x}_{т}} = k_{ш} k_{т.о} S_{в} b_{в} \frac{m_{ш}^{\delta_{в}}}{m_z^{\delta_{в}}} \frac{G}{S}. \quad (12.23)$$

The particular derived $P_{в}^{\bar{x}_{т}}$ is called the gradient of force to the elevator lever on centering and is the important characteristic of the longitudinal static controllability. The gradient of $P_{в}^{\bar{x}}$ is positive, since during the displacement of the center of mass back/ago, the pressing force to lever increases, which corresponds to the deflection of control down. The displacement of

the center of mass back/ago decreases the reserve of centering, it makes the longitudinal static stability of aircraft worse. Consequently, during a decrease in the reserve of the longitudinal static stability on g-force for the balance of aircraft it is necessary to apply the supplementary pressing (positive) force.

In the target/purposes of the more in-depth analysis of the conditions, which affect the value of force feel of elevator, let us determine the pitching moment, which acts on aircraft with the freed steering control. In this case elevator itself "selects" this position, called balancing, in which hinge moment is equal to zero.

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The deflection of control in the balancing position of $\delta_{в.сп}$ can be determined from relationship (12.13), by set/assuming the $m_{ш}=0$:

$$\delta_{\text{в св}} = -\frac{m_{\text{ш}}^{\alpha_{\text{г.о}}}}{m_{\text{ш}}^{\delta_{\text{в}}}}\,\alpha_{\text{г.о}} - \frac{m_{\text{ш}}^{\tau_{\text{в}}}}{m_{\text{ш}}^{\delta_{\text{в}}}}\,\tau_{\text{в}}.$$

The angle of attack of horizontal tail assembly in accordance with formula (9.30) [1] is determined by the dependence:

$$\alpha_{\text{г.о}} = \alpha_0 + \varphi + \left(\frac{1}{c_y^{\alpha}} - D\right) c_y. \qquad (12.24)$$

FOOTNOTE [1]. Value ε_0 in the first approximation, we disregard. ENDFOOTNOTE.

By substituting the value of $\delta_{\text{в.св}}$ taking into account (12.13) and (12.24) in expression (9.51), we will obtain the value of

the coefficient of pitching moment with the freed elevator:

$$m_{z\,\text{св}}=m_{z0\,\text{св}}+m_{z\,\text{св}}^{c_y}c_y, \tag{12.25}$$

where

$$m_{z0\,\text{св}}=m_{z0}+m_z^{\varphi}\varphi+m_{zp}-$$
$$-\frac{m_z^{\delta_\text{в}}}{m_\text{ш}^{\delta_\text{в}}}\left[m_\text{ш}^{\alpha_{\text{г.о}}}(\alpha_0+\varphi)+m_\text{ш}^{\tau_\text{в}}\tau_\text{в}\right]; \tag{12.26}$$

$$m_{z\,\text{св}}^{c_y}=m_z^{c_y}-\frac{m_z^{\delta_\text{в}}}{m_\text{ш}^{\delta_\text{в}}}\,m_\text{ш}^{\alpha_{\text{г.о}}}\left(\frac{1}{c_y^{\alpha}}-D\right). \tag{12.27}$$

The value of $m_{z0\,\text{св}}$ — (coefficient of the pitching moment of

aircraft with the freed elevator with of $c_y=0$— can be changed at will of pilot by a change in the angle of the τ_B, of the deflection of the elevator trim tab and can have any sign.

The value of $m_{z\,\text{сп}}^{c_y}$ is called the criterion for the longitudinal stability of aircraft on g-force with the freed control. Since the partial derivatives of $m_z^{\delta_B}$, $m_{ш}^{\delta_B}$, $m_{ш}^{\dot{\varepsilon}_{\text{г.о}}}$ are negative, if $\left(\frac{1}{c_y^{\tau}}-D\right)>0,$ then

$$\left|m_{z\,\text{сп}}^{c_y}\right|<\left|m_z^{c_y}\right|,$$

i.e. the stability level of aircraft on g-force with the release of control decreases. Taking into account that

$$m_z^{c_y}=\bar{x}_{\text{т}}-\bar{x}_F. \qquad (12.28)$$

it is possible to consider value

$$\bar{x}_{F\,\text{св}} = \bar{x}_F + \frac{m_z^{\delta_\text{в}}}{m_\text{ш}^{\delta_\text{в}}} m_\text{ш}^{\alpha_\text{г.о}} \left(\frac{1}{c_y^\alpha} - D\right) \qquad (12.29)$$

as the longitudinal focus of aircraft with the freed control.

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With the release of elevator, the longitudinal focus of aircraft is displaced forward because of a decrease in the lift increment of horizontal tail assembly with an increase in the angle of attack, since due to the absence of stiffening joint with tail assembly elevator will be deflected upward, stopping "in weathervane" along the flow, which flows around tail assembly. In other words, the basic increase of $\Delta Y_{\text{г.о}}$ will be created by stabilizer, but not all tail assembly. Taking into account relationships (12.28) and (12.29) expression for determining the stability level of aircraft with the freed elevator (12.28) can be written:

$$m_{z\,cn}^{c_y} = \bar{x}_т - \bar{x}_{F\,cв}. \qquad (12.30)$$

Let us determine now the value of force on the steering control of the elevator control. With the balanced pitching moment the required deflection of elevator is assigned by formula (12.6), and the angle of horizontal tail assembly is determined by equality (12.24). By substituting the value of $\delta_в$ and $\alpha_{г.о}$ in expression (12.13) for a hinge-moment coefficient, we will obtain:

$$m_{\text{ш}} = -\frac{m_{\text{ш}}^{\delta_{\text{н}}}}{m_z^{\delta_{\text{н}}}} \left\{ \left[m_z^{c_y} - \frac{m_z^{\delta_{\text{н}}}}{m_{\text{ш}}^{\delta_{\text{н}}}} m_{\text{ш}}^{\alpha_{\text{г.о}}} \left(\frac{1}{c_y^{\alpha}} - D \right) \right] c_y + m_{z0} + \right.$$

$$\left. + m_z^{\varphi}\varphi + m_{zp} - \frac{m_z^{\delta_{\text{н}}}}{m_{\text{ш}}^{\delta_{\text{н}}}} \left[m_{\text{ш}}^{\alpha_{\text{г.о}}} (\alpha_0 + \varphi) + m_{\text{ш}}^{\tau_{\text{н}}} \tau_{\text{н}} \right] \right\},$$

or, taking into account of relationship (12.25), (12.26) and (12.27):

$$m_{\text{ш}} = -\frac{m_{\text{ш}}^{\delta_{\text{н}}}}{m_z^{\delta_{\text{н}}}} m_{z\,\text{св}}. \qquad (12.31)$$

. In the general case the value of $m_{z\,\text{св}}$ is determined by formula (12.25). Among all flight conditions, can be selected this mode/conditions of the balanced flight during which the force on the steering control of control is equal to zero, i.e.,

$$m_z = m_{z\,cn} = 0;\quad m_{ш} = 0. \qquad (12.32)$$

To this flight conditions correspond the completely determined values of the lift coefficient of $c_{yб}$ and velocity head of $q_б$ (bob velocity head).

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Taking into account conditions (12.32) for the mode/conditions of balance for force it is possible to write:

$$m_{z\,cn} c_{yб} + m_{z0\,cn} = 0,$$

By deducting this equality from equality (12.25), we will

obtain:

$$m_{z\,\text{cn}} = m_{z\,\text{cn}}^{c_y} c_y \left(1 - \frac{c_{y6}}{c_y}\right),$$

or taking into account $c_y = \frac{G}{qS}$

$$m_{z\,\text{cn}} = m_z^{c_y} \frac{G}{qS} \left(1 - \frac{q}{q_6}\right).$$

Now expression for a hinge-moment coefficient (12.31) an example is the form:

$$m_{\text{ш}} = -\frac{m_{\text{ш}}^{\delta_{\text{n}}}}{m_z^{\delta_{\text{n}}}} m_z^{c_y} \frac{G}{qS} \left(1 - \frac{q}{q_6}\right). \tag{12.33}$$

By substituting this expression of $m_{\text{ш}}$ in formula for determining force on knob/stick (12.19), we will obtain:

$$P_{\text{в}}=k_{\text{ш}}k_{\text{г.о}}S_{\text{в}}b_{\text{в}}\frac{m_{\text{ш}}^{\delta_{\text{в}}}}{m_z^{\delta_{\text{в}}}}\,\frac{G}{S}\,m_{z\,\text{св}}^{c_y}\left(1-\frac{q}{q_{\text{б}}}\right)$$

or taking into account the value of the gradient of force on centering (12.23)

$$P_{\text{в}}=P_{\text{в}}^{\bar{x}_{\text{т}}}m_{z\,\text{св}}^{c_y}\left(1-\frac{q}{q_{\text{б}}}\right). \qquad (12.34)$$

After expressing velocity heads by the appropriate speeds and mach numbers, we will obtain parabolic communication/connection between force on knob/stick and flight speed (by mach number):

$$P_{\text{в}}=P_{\text{в}}^{\bar{x}_{\text{т}}}m_{z\,\text{св}}^{c_y}\left(1-\frac{V^2}{V_{\text{б}}^2}\right)=P_{\text{в}}^{\bar{x}_{\text{т}}}m_{z\,\text{св}}^{c_y}\left(1-\frac{M^2}{M_{\text{б}}^2}\right). \qquad (12.35)$$

The graph/diagram of this dependence is shown in Fig. 12.12.

Since for the stable aircraft of $P_{в}^{\bar{x}_{т}}>0$ and $m_{z\,cв}^{c_y}<0,$ in formulas (12.34) and (12.35) the constant factor of $P_{в}^{\bar{x}_{т}}m_{z\,cв}^{c_y}<0,$ therefore with an increase in velocity head the pulling force on knob/stick decreases; with $q=q_{б}$ it becomes equal to zero, and then passes over to the pressing force. Regularities (12.34) and (12.35) are disturbed with the onset of shock stall (Fig. 12.13). The displacement of the longitudinal focus back/ago with of $M>M_{кр}$ leads to the appearance of the negative pitching moment, for countering of which the elevator must be deflected upward, i.e., in transonic zone to control, can be applied negative (pulling) force.

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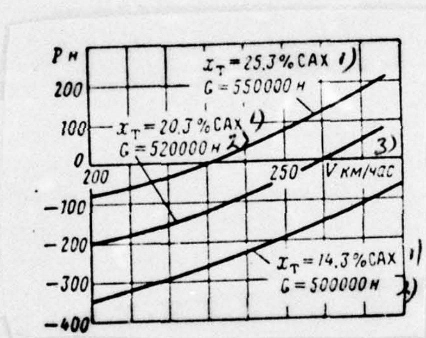
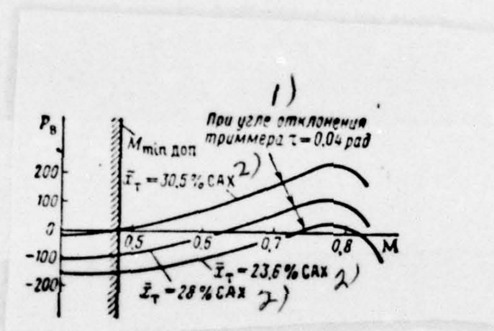


Fig. 12.12. Effect of equivalent airspeed and centering on control force elevator.

Key: (1). MAC. (2). n. (3). km/h.

Fig. 12.13. Effect of mach number centering on control force elevator.

Key: (1). At the angle of deflection of trim tab τ = 0.04 is glad.
(2). MAC.



12.5. Improvement in the characteristic of the longitudinal static controllability.

The handling in pitch of aircraft can be judged from the balancing curve, constructed on the basis of formula (12.34). Since the dependence of $P_{в}$ on q is linear, on diagram $P_{в}=q$ this dependence will be represented by the straight line, passing through the point $(0, P_{в}^{x_{т}} m_{z\,св}^{c_{y}})$ and sloped toward axle/axis q at an angle A whose tangent is equal $\frac{P_{в}^{\bar{x}_{т}} m_{z\,св}^{c_{y}}}{q_{\delta}}$ (Fig. 12.14a). During analysis let us proceed from the condition that the aircraft is assumed to be stable both with "that which was pressed" $(m_{z}^{c_{y}}<0)$, and during the freed $(m_{z\,св}<0)$ control. On the strength of this condition expression

$$\operatorname{tg} A = -\frac{P_{в}^{\bar{x}_{т}} m_{z\,св}^{c_{y}}}{q_{\delta}} > 0, \qquad (12.36)$$

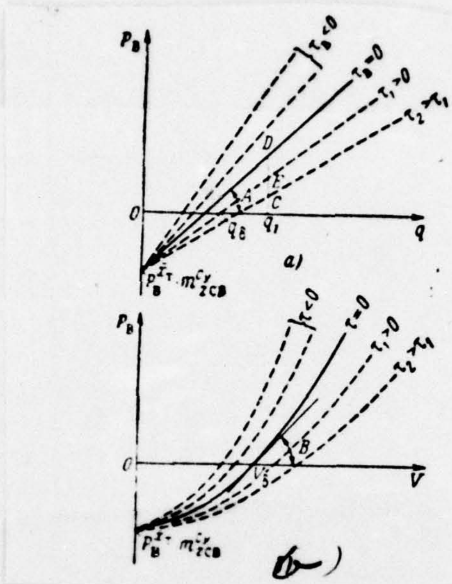
i.e. the straight line composes with axle/axis q positive angle.

If we use for an analysis by formula (12.35) and to construct the graph/diagram of the dependence of $P_{в}=f(V)$, then it will seem the parabola, depicted on Fig. 12.14b. For a stable aircraft the tangent with axle/axis OV will compose angle of B:

$$\mathrm{tg}\, B = -2P_{в}^{\bar{x}_{т}} m_{z\,сн}^{c_y} \frac{V}{V_{б}^{2}} > 0. \qquad (12.37)$$

The given in Fig. (12.14) curve/graphs visually show the character of a change in the force on steering control depending on the value of velocity head or velocity.

Fig. 12.14. Effect of trim tab on control force elevator.



It can seem that the value of force exceeds the established/installed norms and the aircraft control becomes difficult; therefore for providing a controllability necessary to take special measures. As such measures they can serve the application/use of trim tabs, balancers and springs. Let us examine first of all the effect of trim tab on balancing curve. For the different angles of τ_{B} will be different values of m_{z0CB} and, therefore, bob velocity heads or velocities. A change in bob velocity head or velocity will involve change in angles of a and B. As a result let us have a pencil of straight lines and a family of parabolas (Fig. 12.14, dotted line), that are the balancing curves, which remove/take or the increasing forces with the steering control of control.

Thus, for instance, under the conditions of flight appropriate q_1, the force of the P_{B} with of $\tau_{B}=0$ is determined by cut CD, but with τ_2 the force considerably less is determined by cut CE. The installation diagram of balancer and spring is shown in Fig. 12.15. In actuality, they can be established/installed on any component/link of the control system where this will seem convenient designer. Balancer and spring depending on installation diagram can increase or decrease the force on steering control, whereupon during the straight flight of aircraft balancer and spring change force to one and the same value, independent of the velocity of its pressure. In flight

along the curved path of $(n_y \neq 0)$ the force from spring will not be changed, but effect from balancer will be proportional n_y.

Joint effect from the effect of trim tab and spring or balancer is explaind in an example. Let us assume that the balancing curve is such, that $P_{\text{в}}^{q} < 0.$ Then with the aid of balancer or spring we displace it equidistantly down (dotted line in Fig. 12.16), and then, deflecting the trim tab down of $(\tau_{\text{в}} > 0),$ we give to it positive slope/inclination. In Fig. 12.16 slope/inclination is selected so that balancing velocity head would not be changed, although this is not necessarily.

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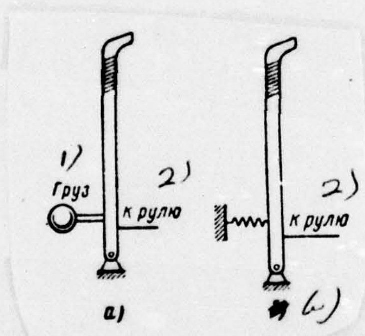
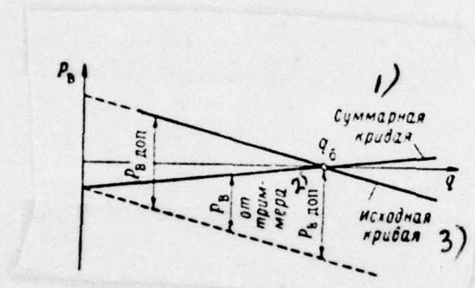


Fig. 12.15. Installation diagram of balancer (a) and of spring (b).

Key: (1). Load. (2). To control.

Fig. 12.16. Correction of balancing diagram with the aid of trim tab and spring or balancer.

Key: (1). Total curve. (2). from trim tab. (3). Initial curve.



Usually is utilized the special adjustable on the earth/ground trim tab.

At transonic and supersonic speeds the center of pressure of elevator is moved back/ago; therefore the force feel of P_B sharply grow/rises. On contemporary aircraft with transonic and, especially, supersonic flight speeds the force feel so big that exceed pilot's physical possibilities. In this case, due to the center-of-pressure travel of elevator over flight mach number it is not possible to select the compensation, acceptable both for subsonic and supersonic speeds. If we select compensation from the calculation for flight at supersonic speeds, then at subsonic speeds will occur overcompensation, which is inadmissible.

A nonuniform change in the force feel of elevator in flight mach number upon transition through the speed of sound makes the characteristics of the longitudinal static controllability worse. For an improvement in the characteristics of the longitudinal static controllability, are applied the amplifiers whose problem first of all entails a decrease in the force feel of elevator.

Questions for repetition.

1. Will the plane possess the longitudinal static stability, if as a result of the load of aircraft the center of mass did prove to be behind the longitudinal focus?

2. How will be changed longitudinal stability factor, if, other conditions being equal, engine nacelles from wing are transferred to aft fuselage section?

3. Will be changed the position of balancing curve for a statically stable aircraft (see Fig. 12.6, $m_z^{c_y}<0$). if the angle of stabilizer setting of that arrange/located in tail aircraft component, is increased on several degrees?

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4. Explain, why for the statically stable aircraft of $\frac{d\delta_n}{dc_y}<0$?

5. Explain, as affects mach number balancing curve on g-force and on speed.

6. Which characteristics of aircraft do change and as, if with the load of aircraft the center of gravity did prove to be in front of maximally bow heaviness?

7. *Explain the designation/purpose* of balancers in the system of controls.

Problem.

A change of the position of center of gravity in aircraft that 134 in flight because of the consumption of fuel/propellant is possible within limits from 30 to 37o/o MAC. To determine the elevator angle with of $\bar{x}_T = 0{,}37$, if it is known that with $\bar{x}_T = 0{,}30$ and at flight speed with mach number = 0.8 elevator angle is equal

$\delta_{в}=0{,}003$ rad It is given: G = 380.000 n, $S_{кр}=115$ m², flight altitude H = 10.000 m, flight speed during a change in the centering remains constant, the elevator-effectiveness derivative during the indicated state of motion is determined by the value of $m_z^{\delta_в}=-0{,}647$. The answer/response: $\delta_в=+0{,}0268$ rad

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Chapter XIII.

LATERAL STATIC STABILITY AND AIRCRAFT HANDLING.

The lateral static stability is determined by the behavior of aircraft with respect to axle/axis Ox_1 (lateral stability) and to axle/axis Oy_1 (is weather cock, or path, stability). The lateral static aircraft handling is determined by the deflection of controls and by the forces, which appear on control levers, for the balancing of the lateral torque/moments in the steady flight.

The need for the balance of the lateral forces and torque/moments, which act on aircraft in uniform rectilinear motion, appears in flight with slip, with the unsymmetric thrust/rod or in the cases of the disturbance/breakdown of geometric, rigid or aerodynamic symmetry. In the curvilinear steady motion the organ/controls of the lateral control can be deflected in the absence of slip, if aircraft rotates relative to axle/axes Ox_1 and Oy_1. Just as in the case of handling in pitch, here important characteristic is the change in the forces on control during the lateral balance of aircraft.

13.1. Lateral static stability.

Lateral stability. If during a change in the roll attitude is a tendency toward the restoration/reduction of its original value, which means, an aircraft it possesses lateral stability, i.e., by static stability on roll attitude. It should be noted that a change in the orientation of the aircraft of relatively earth-based coordinate system does not affect the aerodynamic forces and the torque/moments, which act on it. Therefore the bank of aircraft does not lead to a change in the acting aerodynamic loading and, therefore, to the appearance of a torque/moment, which reduces

starting position. In this case the aircraft in transverse relation is statically neutral.

Then does arise the question: that to understand by lateral stability of aircraft? Let us observe the behavior of aircraft with bank during the small interval of time. For the initial motion let us accept level flight with the zero angle of bank.

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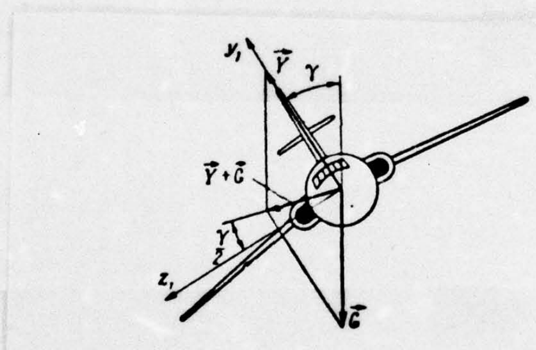

Fig. 13.1. Diagram of the forces, which act on aircraft with bank.

Of the initial motion the sum of the forces, which act on vertical line, is equal to zero, i.e.,

$$\vec{Y}+\vec{G}=0.$$

If under the action of the perturbation factors aircraft inclined itself to angle γ (Fig. 13.1), then, since into zero time the values of the angle of attack and lift do not change, equality $|Y| = |G|$ it is retained.

However, vector sum

$$\vec{Y}+\vec{G}\neq 0.$$

The unbalanced sum of the forces of $(\vec{Y}+\vec{G})$ produces increasing motion, and since it does not lie/rest at the plane, normal to the plane of symmetry, change the angles and attack. The

components of this sum along the axes Oy_1 and Oz_1 are equal to:

$$\left.\begin{array}{l} Y_T = G(1-\cos\gamma), \\ Z_T = G\sin\gamma. \end{array}\right\} \quad (13.1)$$

At small angles of the bank of expression (13.1) takes the form himself

$$Y_T \approx 0; \quad Z_T \approx G\gamma.$$

A change of the angle of attack in this case can be disregarded as small second-order quantity.

For time t after the banking of the plane, it acquires rate of sideslip, equal to

$$V_z = \int_0^t g \frac{Z_1}{G} dt \approx g\gamma t.$$

The slip angle, caused by this motion, is defined as

$$\Delta\beta \approx \mathrm{tg}\,\Delta\beta = \frac{V_z}{V} = \frac{g\gamma t}{V}.$$

Thus, the bank of aircraft is the reason for the emergence of the lateral forward motion to the side of bank and, therefore, by the reason for the appearance of a slip for the omitted wing. If aircraft possesses lateral stability, then with slip appears the rolling moment, directed against the direction of bank, otherwise roll attitude it will grow.

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If $\gamma > 0$, then also $\Delta\beta > 0$, but rolling moment must be negative in order that aircraft would have a tendency toward output/yield from bank. That means aircraft approaches the liquidation of bank, if is fulfilled inequality [1].

$$\frac{\partial M_x}{\partial \beta} = M_x^{\beta} < 0$$

or

$$m_x^{\beta} < 0.$$

FOOTNOTE [1]. In this paragraph the torque/moments are calculated relative to body axes Ox_1i Oy_1; in the designations of torque/moments

index "1" for the simplicity of the writing of formulas is lowered. ENDFOOTNOTE.

It is not difficult to be convinced of the fact that in the case of $m_x^\beta > 0$ the emergent roll attitude will grow/rise, so rolling moment acts to the side of slip (bank). Thus, on the sign of the derivative of m_x^β it is possible to judge presence or absence of lateral stability, and by its value - the stability level or instability. Therefore derived m_x^β is called criterion, or degree, lateral stability. Lateral stability of $(m_x^\beta < 0)$ or instability of the $(m_x^\beta > 0)$ of aircraft can be judged from its behavior with slip. If with slip is a tendency toward bank to the delaying wing, aircraft in transverse relation is stable.

The degree of lateral stability of aircraft and the rolling-moment coefficient (see Chapter X), depends on wing planform, its dimensional characteristics, interference between the wing and the fuselage, the angle of attack and mach number. The special feature/peculiarity of aircraft with sweptback wing is an increase in the degree of lateral stability with an increase of angle of attack. The value of m_x^β can be calculated according to formula (10.19).

Excessively the large stability the reason for the emergence of the lateral oscillating motions, which impede piloting at high angles of attack. To avoid this unpleasant phenomenon to wing is given negative angle dihedral, that it is not possible to consider the radical means, which ensures the necessary degree of lateral stability in all flight conditions, since at low angles of attack $m_{x\psi}^{3}$ does not depend on α (Fig. 13.2).

In flight with the slip with of $M > M_{кр}$ effective mach numbers of pushed forward and delaying wings are different; therefore changes in the $c_y^{\alpha} = f(M)$ for these wings will not be identical. the approximate graph/diagrams of the dependence of $c_y^{\alpha} = f(M)$ for swept wings with relatively thick and fine/thin airfoil/profiles are given in Fig. 13.3. In the range of numbers $M_1 \leqslant M \leqslant M_2$, the lift effectiveness of the delaying wing with relatively thick airfoil/profile are more than pushed forward one along flow, i.e., in this range of mach numbers aircraft in transverse relation is unstable.

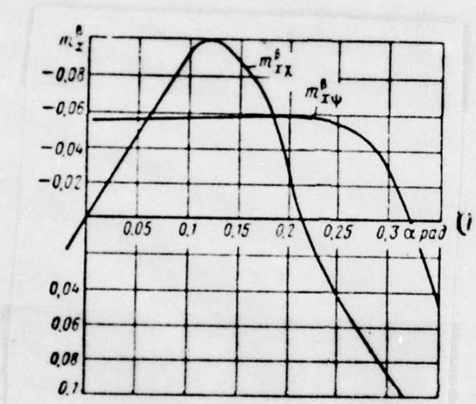

Fig. 13.2. The dependence of $m_{x\psi}^{\beta}$ and $m_{x\chi}^{\beta}$ on angle of attack ($\chi=\pi/4$ rad, λ=5, ψ=+0.1 rad, =5).

Key: (i) rad.

Of wing with relatively fine/thin airfoil/profile, this phenomenon is not observed. In the range of numbers $M_1 < M < M_2$ a difference in the wings on c_y^{β} is small. If we give to wing reverse/inverse dihedral, then the field, where the aircraft is unstable, in the first case will be expanded, and in the second - only it will appear.

The absence of lateral stability - the phenomenon is undesirable, since it brings to the so-called reverse reaction of plane for bank [1].

FOOTNOTE [1]. On this question see 13.2. ENDFOOTNOTE.

On some contemporary passenger aircraft with swept or delta wing, reverse/inverse V is not applied, but the unsatisfactory characteristics of the lateral stability of aircraft during motion with high angles of attack are improved with the aid of automatic means.

Weathercock stability. By weathercock stability is understood the static stability of aircraft on slip angle, i.e., the presence in

the aircraft of tendency toward the preservation/retention/maintaining of the preset angle of slip without the interference in the aircraft control.

In aviation literature this property frequently is called directional stability; however, this term it cannot be recognized successful, since path (course) stability aircraft does not possess.

Weather cock stable aircraft behaves as weathervane - attempts to stop along flow.

It is obvious, aircraft it will possess this property, if with slip appears the static moment of yawing, directed against slip, i.e., the sign of an increase in the torque/moment will be opposite to the sign of an increase in the slip angle, which mathematically is expressed by inequality

$$m_y^\beta < 0.$$

The value of m_y^β is called criterion, or degree, weathercock stability. With $m_y^\beta > 0$ the aircraft is weather cock unstable, with $m_y^\beta = 0$ — is neutral.

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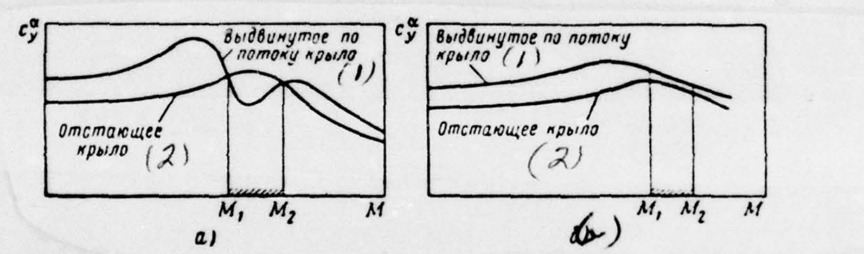

Fig. 13.3. Dependence of $c_y^\alpha = f(M)$ for swept wings with the slip: a) wing with relatively thick airfoil/profile; b) wing with relatively fine/thin airfoil/profile.

Key: (1). Pushed forward along flow wing. (2). Delaying wing.

The degree of weathercock stability is the important characteristic of the aircraft: than is stabler aircraft on slip angle, thereby "more closely it sits" in air, i.e., the lesser it yields to the perturbation effects.

Is especially important the value of the criterion for m_y^β for the passenger aircraft, which has, as a rule, several engines. For an aircraft with the large in module/modulus value of m_y^β the failure of the lateral engine will not lead to difficulties in piloting [1].

FOOTNOTE [1]. This case of flight minutely is examined into 13.3. ENDFOOTNOTE.

The degree of the weathercock stability of aircraft in essence is determined by the torque/moments, created by fuselage and vertical tail assembly, and it depends on wing arrangement relative to fuselage and flight mach number. The value of m_y^β can be calculated according to formula (10.18).

By analogy with stability on the g-force of m_y^β it is possible to express with the aid of the lateral focus. According to formula (10.18)

$$m_y^\beta = -(\bar{x}_\text{т} - \bar{x}_{F\beta})\, c_z^\beta.$$

For passenger aircraft the characteristically even distribution of weight along fuselage is sufficient great lengthening. In connection with this the center of mass of aircraft is arrange/located closely to the middle of the length of fuselage, and therefore from the condition of stabilization for g-force wing must be related back/ago. This layout contributes to an increase in the destabilizing torque/moment of fuselage and to a reduction/descent in the weathercock stability. The only real construction means for an increase in the weathercock stability is an increase in the fin-and-rudder area.

For passenger aircraft for the target/purposes of the better/best use of a fuselage, is applied high or low wing arrangement on fuselage.

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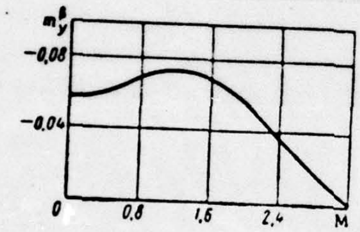

Fig. 13.4. Dependence of $m_y^\beta = f(M)$

As it was noted in chapter X, with the high wing arrangement, the lateral taper in the field of tail assembly, caused by wing-root interference effect, lowers the effectiveness of vertical tail assembly and, which means, the degree of weathercock stability. Therefore the ensurence of the necessary degree of weathercock stability for such aircraft requires even a larger increase in the vertical tail assembly, setting up of fin both on top and from below fuselage, the setting up of supplementary washers on horizontal tail assembly, etc.

From this viewpoint most successful is the layout with low wing arrangement, which facilitates an increase in the effectiveness of vertical tail assembly.

Of supersonic aircraft with an increase in Mach number, decreases the effectiveness of vertical tail assembly (falls $c^{\beta}_{z\,\text{в.о}}$), while the effectiveness of fuselage (derived $c^{\beta}_{z\,\Phi}$) changes weakly. This leads to a decrease in the stabilizing moment of tail assembly; with certain mach number it becomes equal to the destabilizing torque/moment of fuselage. With mach numbers, that exceed this value, aircraft loses weathercock stability (Fig. 13.4).

Section -

§ 13.2. Balance of aircraft in the rectilinear steady flight WITH slip.

In the rectilinear steady flight in a slip to the aircraft, which possesses the lateral static stability, will act statically the lateral forces and torque/moments, in particular the lateral aerodynamic force, the yawing moment, directed to the side of slip, and the moment of roll, directed to the side of the delaying wing (Fig. 13.5). The force intensities and torque/moments are determined from the formulas of chapter X.

For the balance of these torque/moments, it is necessary to incline aircraft and to deflect ailerons and rudder. If we disregard the yawing moment, created by ailerons during their deviation (see 10.2), then the moment conditions of equilibrium and lateral forces can be written in the form:

$$\left.\begin{aligned} M_x &= M_x^{\beta}\beta + M_x^{\delta_{\text{э}}}\delta_{\text{э}} + M_x^{\delta_{\text{н}}}\delta_{\text{н}} = 0, \\ M_y &= M_y^{\beta}\beta + M_y^{\delta_{\text{н}}}\delta_{\text{н}} = 0, \\ Z &= G\sin\gamma + Z^{\beta}\beta + Z^{\delta_{\text{н}}}\delta_{\text{н}} = 0, \\ Y &- G\cos\gamma = 0. \end{aligned}\right\} \qquad (13.3)$$

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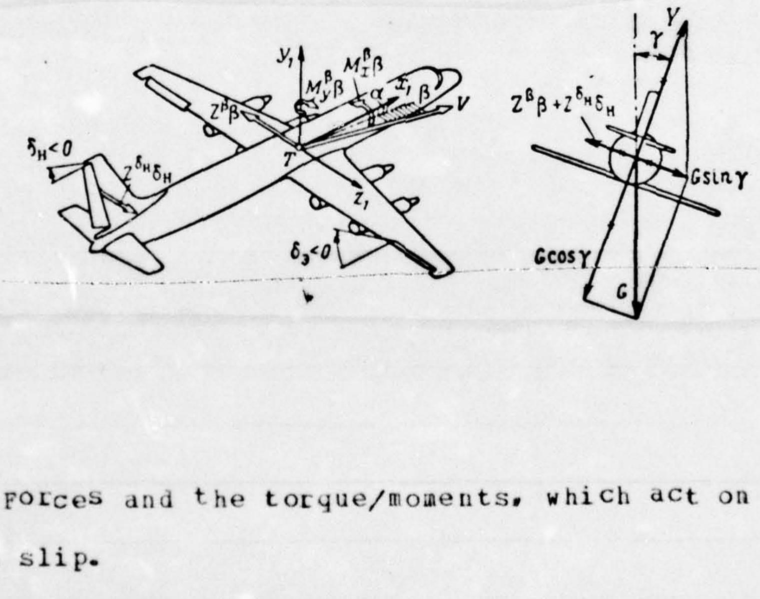

Fig. 13.5. Forces and the torque/moments, which act on aircraft in flight with slip.

After dividing the first two equation in qSl, and the third on qS and after eliminating G from the third equation, we will obtain the equilibrium conditions, written for the force coefficients and torque/moments:

$$\left.\begin{aligned} m_x &= m_x^{\beta}\beta + m_x^{\delta_э}\delta_э + m_x^{\delta_н}\delta_н = 0, \\ m_y &= m_y^{\beta}\beta + m_y^{\delta_н}\delta_н = 0, \\ c_z &= c_y \operatorname{tg}\gamma + c_z^{\beta}\beta + c_z^{\delta_н}\delta_н = 0. \end{aligned}\right\} \quad (13.4)$$

By defining from the obtained system of equations roll attitudes, the deviation of ailerons and rudder as functions of angle β, we will obtain the balancing values of these angles, necessary for the fulfillment of the rectilinear steady flight in a slip:

$$\left.\begin{aligned}
&\gamma \approx \operatorname{tg}\gamma = -\frac{c_z^{\beta}}{c_y}\left(1-\frac{c_z^{\delta_н}}{c_z^{\beta}}\,\frac{m_y^{\beta}}{m_y^{\delta_н}}\right)\beta,\\
&\delta_э = \frac{1}{m_x^{\delta_э}}\left(\frac{m_x^{\delta_н}m_y^{\beta}}{m_y^{\delta_н}}-m_x^{\beta}\right)\beta,\\
&\delta_н = -\frac{m_y^{\beta}}{m_y^{\delta_н}}\,\beta.
\end{aligned}\right\} \quad (13.5)$$

For a statically stable aircraft it is necessary to create bank to the side of the pushed forward wing, and rudder and ailerons to deflect to the side slips, i.e., ailerons during the pushed forward relative to flow wing are deflected upward, but on that which delay they are deflected down.

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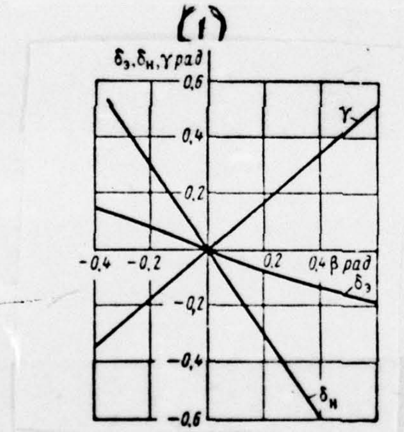
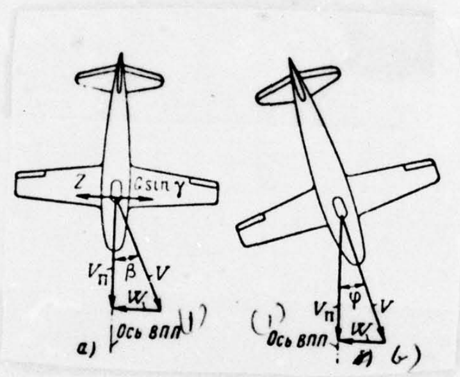


Fig. 13.6. Approximate balancing diagram in flight with slip.

Key: (1). ~~is glad~~. rad

Fig. 13.7. Diagram of the prelanding motion in the presence of the cross wind: a) with slip; b) with advance/prevention along course.

Key:. (1). Axle/axis runways.



The balancing diagram of this flight is constructed in the form of the dependences of $\delta_н=f(\beta)$, $\delta_э=f(\beta)$ and $\gamma = f(\beta)$. Approximate balancing diagram for a statically stable in the lateral relation aircraft is given to Fig. 13.6. On passenger aircraft the need flight in a slip appears comparatively rarely. This flight can be utilized during crosswind landing or in flight with unsymmetric thrust/rod.

Cross wind produces airplane drift relative to takeoff and landing strip; in order that motion would occur along band runways, necessary the agreement of the vector of the ground speed of aircraft with the axle/axis of band, which is reached by flight in a slip (Fig. 13.7a) or with advance/prevention along course (Fig. 13.7b). It is possible also to make motion with slip and simultaneous advance/prevention along course. In flight with slip, the longitudinal axis of aircraft is directed along landing strip, and the slip angle is determined by equality

$$\beta \approx \sin\beta = \frac{W}{V}.$$

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In this case the required deviations of the organ/controls of the lateral control can be found from conditions (13.5). For the preservation/retention/maintaining of the straightness of motion, the aircraft one should incline for leeward wing. Before the contact by wheels runways, the aircraft is derive/concluded from bank by the deviation of steering control in the direction of wind. If the degree of lateral stability is great, then required the deviation of ailerons for the balance of the torque/moment, caused by slip, will be considerable.

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On aircraft with swept or deltas, the degree of lateral stability increases with an increase of c_y. Therefore for such aircraft during crosswind landing, the required aileron deflections can turn out to be unacceptable.

If the prelanding flight with cross wind is made with advance/prevention on course of angle ϕ= W/V, i.e., the nose of aircraft is expand/developed to the side of wind, then slip is absent, and therefore there is no need for for the balance of the

lateral forces and torque/moments; however before the contact by the wheels of the Earth aircraft must be expand/developed on band.

In flight during the deflection of rudder, appears the side slip, opposite to the direction of the rotation of control. In this case on aircraft, acts the moment of roll, determined by the rolling-moment coefficient:

$$m_x = m_x^{\beta}\beta + m_x^{\delta_н}\delta_н.$$

Let us assume that the bank of aircraft under the action of the moment of roll occurs at the establish/installed angular velocity, determined by the moment condition of equilibrium relative to axle/axis Ox_1. Then during the roll is occur relationship

$$m_x^{\omega_x}\omega_{x\,уст} + m_x^{\beta}\beta + m_x^{\delta_н}\delta_н = 0. \tag{13.6}$$

After eliminating β from equality (13.6) with the aid of the third equation of system (13.5), we will obtain the following expression for the establish/installed angular rate of rotation of the relatively longitudinal axis:

$$\omega_{x\,\text{уст}} = -\frac{1}{m_x^{\omega_x}}\left(-m_x^{\beta}\frac{m_y^{\delta_\text{н}}}{m_y^{\beta}} + m_x^{\delta_\text{н}}\right)\delta_\text{н}. \qquad (13.7)$$

From function (13.7) it follows that with $m_x^{\omega_x}<0$ and the negative value of the expression, which stands in brackets, and the positive deviation of rudder (to the right) the aircraft will heel for left wing $(\omega_{x\,\text{уст}}<0)$, i.e., will occur reverse reaction for bank to the deflection of rudder. The negative value of expression

$$-m_x^{\beta}\frac{m_y^{\delta_\text{н}}}{m_y^{\beta}} + m_x^{\delta_\text{н}} < 0 \qquad (13.8)$$

or

$$m_x^{\beta} > \frac{m_y^{\beta}}{m_y^{\delta_{н}}} m_x^{\delta_{н}} \qquad (13.9)$$

places certain conditions on the coefficients of derived aerodynamic couples. Since the derivatives of m_y^{β}, $m_y^{\delta_{н}}$, $m_x^{\delta_{н}}$ are less than zero, condition (13.9) can be made even with $m_x^{\beta} < 0$. i.e., stable in transverse relation aircraft.

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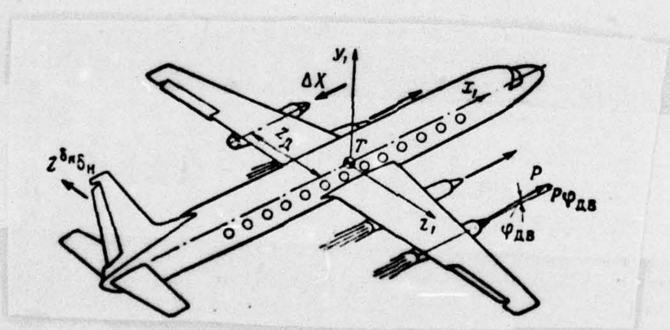

Fig. 13.8. Forces, which act on aircraft in flight power-off.

Thus, statement, that the reverse reaction for bank to the deflection of rudder has a place for an unstable in transverse relation aircraft (§ 13.1), not it is entirely accurate. Reverse reaction for bank to the deflection of control is observed on the aircraft, which have the low degree of lateral stability, or neutral in transverse relation $(m_x^\beta = 0)$.

/ 13.3. Balance of aircraft with failure of one of the engines.

The engine failure on aircraft with several power plants is the perturbation factor, which produces the unsteady disturbed motion, in process of which change the kinematic parameters of motion. With failure of one of the engines, the thrust/rod of left and right-side engines is dissimilar, and therefore this flight frequently is called flight with unsymmetric thrust/rod.

By disregarding the transfer disturbed motion and the actions of pilot on the target/purposes of the stabilization of position, let us examine the steady rectilinear flight of aircraft with failed engine. Flight can be made either with slip or with bank, or with the fact and other simultaneously. If slip is absent, then to aircraft will

act the lateral torque/moments, caused by asymmetry of thrust with respect to fore-and-aft plane. The yawing moment is created by thrust P of the engine, symmetrical to that which was failed, and by the force of supplementary resistance ΔX failed engine.

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The heeling torque/moment is created transverse by the thrust component of engine on, and for aircraft with TVD [ТВД - turboprop engine] or PD [ПД - instrument panel] - also by the lift, which appears as a result of the unsymmetric blowout of wing by screw/propellers. These torque/moments according to Fig. 13.8 can be determined by formulas

$$\left.\begin{array}{l} M_{yp}=-(P+\Delta X)Z_{\text{д}}, \\ M_{xp}=P\varphi_{\text{д}}z_{\text{д}}+M_{x\,\text{обд}}, \end{array}\right\} \qquad (13.10)$$

where the $z_{\text{д}}$ - the coordinate of the axle/axis of failed engine.

By passing over to the force coefficients and torque/moments, we will obtain

$$\left.\begin{aligned} m_{yp} &= -\frac{1}{2}(c_p+\Delta c_x)\,\bar{z}_{\text{д}}, \\ m_{xp} &= \frac{1}{2}\,c_p\varphi_{\text{д}}\bar{z}_{\text{д}}+m_{x\,\text{обд}}, \end{aligned}\right\} \quad (13.11)$$

where

$$\bar{z}_{\text{д}}=\frac{2z_{\text{д}}}{l},\quad c_p=\frac{P}{qS}.$$

Now under conditions of the equilibrium of the lateral forces and torque/moments (13.4) will appear the lateral torque/moments from unsymmetric thrust/rod (13.11):

$$\left.\begin{aligned}
&m_{xp}+m_x^{\delta_э}\delta_э+m_x^{\delta_н}\delta_н+m_x^{\beta}\beta=0,\\
&m_{yp}+m_y^{\delta_н}\delta_н+m_y^{\beta}\beta=0,\\
&c_y\,\mathrm{tg}\,\gamma+c_z^{\delta_н}\delta_н+c_z^{\beta}\beta=0.
\end{aligned}\right\} \qquad (13.12)$$

After solving system of equations (13.12), let us determine the required for balance roll attitudes and the rudder angles and ailerons; set/assuming $m_{xp}\approx 0$, we will obtain

$$\left.\begin{aligned}
&\gamma\approx \operatorname{tg}\gamma=\frac{m_{yP}}{m_y^{\delta_н}}\,\frac{c_z^{\delta_н}}{c_y}+\frac{1}{c_y}\left(c_z^{\delta_н}\,\frac{m_y^{\beta}}{m_y^{\delta_н}}-c_z^{\beta}\right)\beta,\\
&\delta_н=-\frac{m_{yP}}{m_y^{\delta_н}}-\frac{m_y^{\beta}}{m_y^{\delta_н}}\,\beta,\\
&\delta_э=\frac{m_x^{\delta_н}}{m_x^{\delta_э}}\,\frac{m_{yP}}{m_y^{\delta_н}}+\frac{1}{m_x^{\delta_э}}\left(m_x^{\delta_н}\frac{m_y^{\beta}}{m_y^{\delta_н}}-m_x^{\beta}\right)\beta.
\end{aligned}\right\}\qquad(13.13)$$

In flight without slip with unsymmetric thrust/rod, the trim angles are determined from formulas (13.13), in which one should place $\beta = 0$.

If $|z_д|$ is small (engine off is arranged closely the plane of symmetry), then the required deflection of rudder will be low. But if engines are arranged at considerable distance from the plane of symmetry, the balancing deflection of rudder is obtained large.

Fig. 13.9. Approximate balancing diagram in flight with unsymmetric thrust/rod (is included right-side engine).

Key: (1). [redacted] rad

Fig. 13.10. Projections of angular velocity on the axis of body coordinate system (standard rate turn).

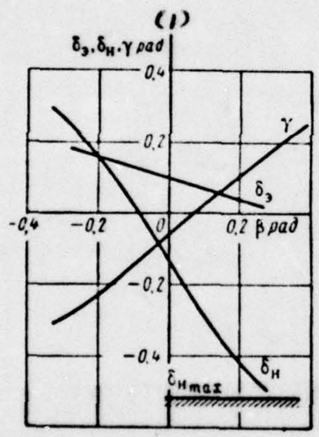

Fig. 13.9.

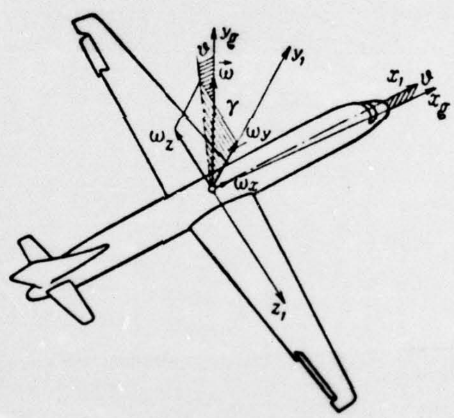

Fig. 13.10.

In order to avoid the high expenditure of rudder in balance, it is better flight to make with side slip, opposite to failed engine. Then the static moment, caused by slip, partially balances torque/moment from the engine thrust and the required deflection of rudder it decreases. It is obvious, in this case, the required aileron angle and the required roll attitude they increase.

Typical for a passenger aircraft balancing diagram in the case of flight with the inoperative right-side engine is given in Fig. 13.9.

It is not difficult to test that in flight with side slip with engines on the required rudder angle decreases, but grow/rise the aileron angle and the roll attitude in comparison with coordinated flight. This flight is made with aircraft with large $|z_{\pi}|$.

And finally flight with unsymmetric thrust/rod without bank can be satisfied with side slip with the shut-down engine. This flight frequently is applied in severe weather conditions. The required angles of deflection of controls and the required slip angle in this case are determined from relationship (13.13) with $\gamma = 0$.

[redacted]

[redacted]

[redacted]

[redacted]

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§ 13.4. Balance of forces and torque/moments in the establish/installed curvilinear motion.

In curvilinear motion even in the absence of slip appears the need for the balance of forces and moments with respect to all three axle/axes of aircraft, if only the plane of the motion of the center of mass of aircraft it does not coincide with the plane of symmetry.

This need is caused by the rotation/revolution of aircraft relative to axle/axes Ox_1, Oy_1 and Oz_1, in consequence of which on it they act the damping and spiral moments, without balancing of which is impossible the execution of the assigned curvilinear motion.

Let us examine as an example the balance of forces and torque/moments with standard rate turn. In this case the aircraft rotates relative to vertical axle/axis at constant angular velocity ω (Fig. 13.10); the center of mass moves in horizontal plane, and the plane of symmetry is sloped toward it at an angle γ. The components of angular velocity ω on the basis of body coordinate system are respectively equal to:

$$\left.\begin{aligned} \omega_x &= \omega \sin \vartheta, \\ \omega_y &= \omega \cos \vartheta \cos \gamma, \\ \omega_z &= -\omega \cos \vartheta \sin \gamma. \end{aligned}\right\} \quad (13.14)$$

If we are restricted to the low pitch angles (with standard rate turn $\vartheta = \alpha$), of formula (13.14) it is possible to write in the form

$$\left.\begin{aligned} \omega_x &\approx \omega\vartheta, \\ \omega_y &\approx \omega\cos\gamma, \\ \omega_z &\approx -\omega\sin\gamma. \end{aligned}\right\} \quad (13.15)$$

The required control deflections for the balance of the acting on aircraft torque/moments we find from conditions

$$\left.\begin{aligned} m_x &= m_x^{\omega_x}\omega_x + m_x^{\omega_y}\omega_y + m_x^{\delta_н}\delta_н + m_x^{\delta_э}\delta_э = 0, \\ m_y &= m_y^{\omega_x}\omega_x + m_y^{\omega_y}\omega_y + m_y^{\delta_н}\delta_н, \\ m_z &= m_{z0} + m_z^{c_y}c_y + m_z^{\omega_z}\omega_z + m_z^{\delta_в}\delta_в = 0, \end{aligned}\right\} \quad (13.16)$$

where

$$c_y = \frac{n_y G}{qS}.$$

The value of the required thrust of engine, necessary for determining M_z, find from condition

$$\sum X = 0.$$

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Since motion establish/installed ($\alpha =$ const),

$$M_z^{\dot{}} = 0.$$

By taking into account value

$$\omega=\frac{g\sqrt{n_y^2-1}}{V}, \quad \frac{1}{n_y}=\cos\gamma$$

and dependence (13.15), after the solution to equations (13.16) relative to the angles of deflection of controls we will obtain their following values:

$$\left.\begin{aligned}
\delta_э &= -\frac{g\sqrt{n_y^2-1}}{Vm_x^{\delta_э}}\left[\left(m_x^{\omega_x}-\frac{m_x^{\delta_н}}{m_y^{\delta_н}}m_y^{\omega_x}\right)\vartheta+\right.\\
&\qquad\left.+\left(m_x^{\omega_y}-\frac{m_x^{\delta_н}}{m_y^{\delta_н}}m_y^{\omega_y}\right)\frac{1}{n_y}\right],\\
\delta_н &= -\frac{g\sqrt{n_y^2-1}}{Vm_y^{\delta_н}}\left(m_y^{\omega_x}\vartheta+\frac{1}{n_y}m_y^{\omega_y}\right),\\
\delta_в &= -\frac{1}{m_z^{\delta_в}}\left(m_{z0}+m_z^{c_y}c_y-m_z^{\omega_z}g\frac{n_y^2-1}{n_yV}\right).
\end{aligned}\right\} \quad (13.17)$$

With the right bank/turn the signs of $\delta_э$ and δ_H are changed by opposite, while the sign of δ_B does not depend on the direction of turn.

Thus, during the execution of standard rate turn elevator is deflected additionally upward (in comparison with horizontal by flight at the same velocity), rudder and ailerons, as a rule, are

deflected to the side of bank/turn (bank).

The required control deflections with standard rate turn are not contained by the values, determined by formulas (13.17). Supplementary deflections will be required for the balance of gyroscopic torque/moments and torque/moments from the Coriolis forces, which act on those which are moving relative to the aircraft of mass (fuel/propellant on conduit/manifolds, air in air intakes TRD [turbojet engine], gases, etc.).

With the steady turn SOS [self-organizing system] with slipping of the balancing aileron angles and rudder are determined in the form of the sum of the values, determined by expressions (13.17) and (13.4).

§13.5. Hinge moments and forces in the thrust/rods of the organ/controls of the lateral control.

The forces, applied by pilot to steering control and pedals for deflection or retention of ailerons and rudder in the deflected for

the balance of the lateral torque/moments position, are proportional to the moments of the aerodynamic forces, which act on control and ailerons relative to the joints of the suspension of these controls.

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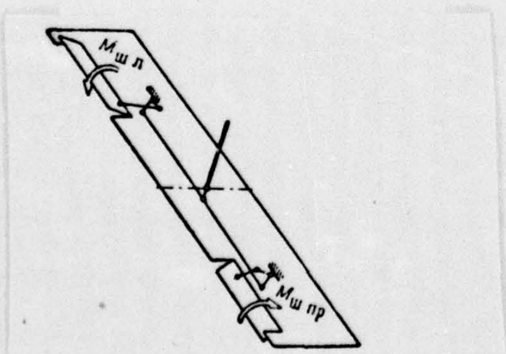

Fig. 13.11. scheme of control of ailerons.

By hinge moments are defined also the forces, which act in control rods; therefore these torque/moments must be known both during the automatic control or with application/use of booster drives and with pilot's direct participation in the lateral control.

During the aileron deflection of positive angle (Fig. 13.11) (upward - for left and down - for right ailerons) relative to the joints of their suspension will act the aerodynamic couples: negative - for right and positive - for left ailerons. These torque/moments can be determined by the formulas of similarity

$$M_{\text{ш.пр}}=m_{\text{ш.пр}}qS_{\text{э1}}b_{\text{э}},$$
$$M_{\text{ш.л}}=m_{\text{ш.л}}qS_{\text{э1}}b_{\text{э}},$$

where the $S_{\text{э1}}, b_{\text{э}}$ — area and aileron chord.

From fundamental scheme of control of ailerons, it is evident

that the force, which acts on steering control (or in control rods), depends on a difference in the hinge moments, i.e., on value

$$M_{ш.э}=M_{ш.пр}-M_{ш.л}, \tag{13.18}$$

and is equal

$$P_э=k_{ш.э}M_{ш.э}. \tag{13.19}$$

The gear ratio of force from aileron to steering control or to control actuator

$$k_{ш.э}=\frac{d\delta_э}{dx},$$

where x - the linear displacement of steering control or thrust/rod, that transmit force to control actuator, depending on that, is determined force on steering control or in thrust/rod.

The force of $P_э>0$, if it is directed to the side of the right wing. The hinge moment of $M_{ш.э}$ is accepted to relate to the total area of ailerons, i.e.,

$$M_{ш.э}=m_{ш.э}qS_эb_э, \qquad (13.20)$$

where the $S_э=2S_{э1}$.

The hinge-moment coefficient of $m_{ш.э}$ according to formula (13.18) is equal to

$$m_{ш.э}=0,5(m_{ш.пр}-m_{ш.л}). \qquad (13.21)$$

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The coefficient of $m_{ш.э}$ depends on the angles of attack and aileron deflection, and also from Mach number of flight. At low angles of attack dependence linear and its terms can be represented in the form

$$\left.\begin{aligned} m_{ш.пр} &= m^{\alpha}_{ш\,n\,пр}\alpha_{n\,пр} + m^{\delta_э}_{ш\,n\,пр}\delta_{э.пр}, \\ m_{ш.л} &= m^{\alpha}_{ш\,n\,л}\alpha_{n\,л} - m^{\delta_э}_{ш\,n\,л}\delta_{э.л}, \end{aligned}\right\} \quad (13.22)$$

where the $m^{\alpha}_{ш\,n\,л}, m^{\alpha}_{ш\,n\,пр}$ — derivatives of the hinge-moment coefficients of ailerons, determined in the cross sections, normal to the aerodynamic center line of wing; $\alpha_{nпр}, \alpha_{nл}$ are angles of attack in these cross sections.

If the aileron deflection upward and downward is equal, then $\delta_{э.пр} = \delta_{э.л}$. Subsequently we will be restricted to the examination of this case. By taking into account communication/connection between the angle of attack of α_n chordwise, normal to aerodynamic center line, and the angle of attack α chordwise, parallel to the plane of

the symmetry of sweptback wing, and also of the relationship of $q_n = q\cos^2\chi$ and $\delta_{\text{э}n} = \delta_{\text{э}}$, from formulas (13.22) we will obtain the following values of the hinge-moment coefficients of ailerons in flight with the slip:

$$m_{\text{ш.пр}} = \frac{m_{\text{ш}}^{\alpha}}{\cos\chi}\,\alpha\cos(\chi-\beta) + \frac{m_{\text{ш}}^{\delta_{\text{э}}}}{\cos^2\chi}\,\delta_{\text{э}}\cos^2(\chi-\beta),$$

$$m_{\text{ш.л}} = \frac{m_{\text{ш}}^{\alpha}}{\cos\chi}\,\alpha\cos(\chi+\beta) - \frac{m_{\text{ш}}^{\delta_{\text{э}}}}{\cos^2\chi}\,\delta_{\text{э}}\cos^2(\chi+\beta).$$

Substituting the value of $m_{\text{ш.пр}}$ and $m_{\text{ш.л}}$ in formula (13.21) and being restricted to low values β, (sine $\beta = \beta$, cos $\beta = 1$), we will obtain for $m_{\text{ш.э}}$ expression

$$m_{\text{ш.э}} = m_{\text{ш}}^{\delta_{\text{э}}}\left(\frac{m_{\text{ш}}^{\alpha}}{m_{\text{ш}}^{\delta_{\text{э}}}}\,\alpha\beta\,\mathrm{tg}\,\chi + \delta_{\text{э}}\right). \qquad (13.23)$$

From this expression it follows that, in the first place, in flight with slip in the aileron control rods they will be to act

forces even in their neutral position of $(\delta_{э}=0)$, that is not observed on aircraft with unswept wing; in the second place, the forces, which act in the aileron control rods in the absence of slip, they do not depend on angle of attack; on aircraft with unswept wing and in flight with slip occurs the invariance of hinge moment of the aerolons and, consequently, also the invariance of forces on angle of attack.

The coefficients of $m_{ш}^{\alpha}$ and $m_{ш}^{\delta_{э}}$ can be designed with the aid of formulas (12.14) at (12.15), if $S_{0.K}$ and S_{B} they are replaced $S_{0.K.э}$ and $S_{э}$.

The values of $m_{ш}^{\delta_{э}}$ and $m_{ш}^{\alpha}$ significantly change with an increase of number M. An example of the dependence of these values from Mach number is given in Fig. 13.12.

Fig. 13.12. Dependence of $m_{ш}^{\delta_э}$ and $m_{ш}^{\alpha}$ on Mach number.

Fig. 13.13. Scheme of control of rudder.

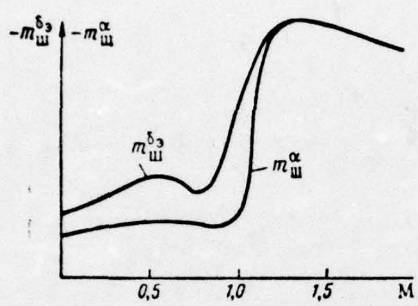

Fig. 13.12.

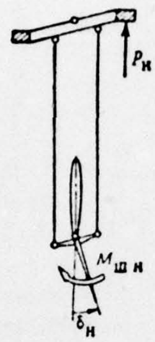

Fig. 13.13.

The forces, which act on the control pedal of rudder, are determined by hinge moment. With the deviation of the rudder to the right of $(\delta_H > 0)$, appears the negative hinge moment of $M_{ш.H}$ (Fig. 13.13), for balancing of which it is necessary to make effort to the right pedal. It is accepted to consider the pressing force by the right pedal positive. Then, on the strength of the virtual work principle, we obtain

$$P_{н} = -k_{ш.н} M_{ш.н}. \tag{13.24}$$

The gear ratio of force from control to pedal

$$k_{ш.н} = \frac{d\delta_{н}}{dx_{н}},$$

where the dx_H - the linear displacement of pedal.

According to the theory of aerodynamic similarity the hinge

moment of $M_{ш.н}$ can be represented in the form

$$M_{ш.н}=m_{ш.н}k_{в.о}qS_{н}b_{н}, \qquad (13.25)$$

where the $S_{н}$, $b_{н}$ - area and the chord of rudder; $k_{в.о}$ q are velocity head in the field of vertical tail assembly.

Hinge-moment coefficient $m_{ш.н}$ depends on the effective angle, rudder angle, Mach number of flight. With low β the dependence of $m_{ш.н}(\beta, \delta_{н})$ can be written in the form

$$m_{ш.н}=m_{ш}^{\delta_{н}}\delta_{н}+m_{ш}^{\beta}\beta_{в.о}, \qquad (13.26)$$

where the $\beta_{в.о}=\beta-\varepsilon_{z}=\beta\left(1-\frac{d\varepsilon_{z}}{d\beta}\right)$; ε_{z} - angle of the lateral taper in the area of the vertical tail assembly.

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The coefficients of $m_{ш}^{\delta_{н}}, m_{ш}^{\beta}$ can be determined experimentally or the calculation according to the approximation formulas.

The character of a change in the $m_{ш}^{\delta_{н}}$ and $m_{ш}^{\beta}$ in Mach numbers is analogous to the dependences of $m_{ш}^{\delta_{э}} = f(M)$ and $m_{ш}^{\alpha} = f(M)$, shown in Fig. 13.12.

A decrease in the hinge moments (forces) of ailerons and rudder just as for an elevator, is achieved the use of aerodynamic balance. On the ailerons of contemporary high-speed aircraft, widely is applied internal compensation. On ailerons and rudders, directly controlled by pilot, is establish/installed the aerodynamic trim tab, which makes it possible to remove/take forces from steering control and pedals in the assigned flight conditions.

§ 13.6. Balancing curves on forces in the thrust/rods of the lateral control.

The values of the forces, which act on steering control and pedals at direct control of these organ/controls, can be written with the aid of formulas (13.19), (13.23), (13.24) and (13.26) in the form

$$\left.\begin{aligned} P_{н} &= -k_{ш.н} S_{н} b_{н} k_{н.о} q \left(m_{ш}^{\delta_{н}} \delta_{н} + m_{ш}^{\beta_{н.о}} \beta_{н.о}\right), \\ P_{э} &= k_{ш.э} S_{э} b_{э} q \left(m_{ш}^{\delta_{э}} \delta_{э} + m_{ш}^{\alpha} \alpha \beta \operatorname{tg} \gamma\right). \end{aligned}\right\} \quad (13.27)$$

Let us examine the lateral static stability with the release of the rudder and ailerons.

If rudder is freed, then hinge-moment coefficient * $m_{ш.н} = 0$; therefore in accordance with formula (13.26) rudder spontaneously deflected in angle

$$\delta_{\text{н.св}} = -\frac{m_{\text{ш}}^{\beta}}{m_{\text{ш}}^{\delta_{\text{н}}}}\beta_{\text{в.о}} = -\frac{m_{\text{ш}}^{\beta}}{m_{\text{ш}}^{\delta_{\text{н}}}}\left(1 - \frac{d\varepsilon_z}{d\beta}\right). \qquad (13.28)$$

Substituting this value of $\delta_{H.CB}$ in expression for the yawing-moment coefficient (13.4), we will obtain

$$m_{y\,\text{св}} = m_y^{\beta}\beta - \frac{m_{\text{ш}}^{\beta}}{m_{\text{ш}}^{\delta_{\text{н}}}} m_y^{\delta_{\text{н}}}\left(1 - \frac{d\varepsilon_z}{d\beta}\right)\beta,$$

whence the degree of the weathercock stability of aircraft with the freed rudder is equal to:

$$m_{y\,\text{св}}^{\beta} = m_y^{\beta} - m_y^{\delta_{\text{н}}}\frac{m_{\text{ш}}^{\beta}}{m_{\text{ш}}^{\delta_{\text{н}}}}\left(1 - \frac{d\varepsilon_z}{d\beta}\right). \qquad (13.29)$$

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The release of rudder will lead also to a change in the degree of lateral stability whose value can be determined, after substituting the value of δ_H from formula (13-28) into relationship (13-4) and by set/assuming in this case, that the ailerons are neutral:

$$m^{\beta}_{x\,\text{св}}=m^{\beta}_{x}-m_{x}^{\delta_{\text{н}}}\frac{m^{\beta}_{\text{ш}}}{m^{\delta_{\text{н}}}_{\text{ш}}}\left(1-\frac{d\varepsilon_z}{d\beta}\right). \tag{13.30}$$

Since $m_{\text{ш}}^{\beta}<0,\ m_{\text{ш}}^{\delta_{\text{н}}}<0,\ m_{x}^{\delta_{\text{н}}}<0,\ \frac{d\varepsilon_z}{d\beta}<1,$ that $m_{x\,\text{св}}^{\beta}>m_{x}^{\beta},$ i.e. the release of rudder lowers lateral stability of aircraft.

By substituting in first relationship (13.27) the value of δ_H from (13.5) and by taking into account relationship (13.29), we will obtain expression for determining pedal force of control of rudder:

$$P_{\text{н}}=k_{\text{ш.н}}S_{\text{н}}b_{\text{н}}k_{\text{в.о}}q\frac{m_{\text{ш}}^{\delta_{\text{н}}}}{m_{y}^{\delta_{\text{н}}}}m_{y\,\text{св}}^{\beta}\beta. \qquad (13.31)$$

Hence it follows that the force, that act on pedal in flight with slip, is proportional to the degree of the weathercock stability of aircraft with the freed rudder and to slip angle.

By discussing analogously, from relationship (13.23) it is possible to determine the angle to which will be deflected with slip the freed ailerons:

$$\delta_{э.сп} = -\frac{m_{ш}^{\alpha}}{m_{ш}^{\delta_э}} \alpha\beta \,\mathrm{tg}\,\chi. \qquad (13.32)$$

in this case to aircraft will act the moment of roll

$$m_{x\,сп} = m_x^{\beta}\beta - m_x^{\delta_э}\frac{m_{ш}^{\alpha}}{m_{ш}^{\delta_э}}\alpha\beta\,\mathrm{tg}\,\chi.$$

Hence the degree of lateral stability with the freed ailerons can be written in the form

$$m_{x\,сп}^{\beta} = m_x^{\beta} - m_x^{\delta_э}\frac{m_{ш}^{\alpha}}{m_{ш}^{\delta_э}}\alpha\,\mathrm{tg}\,\chi. \qquad (13.33)$$

Thus, the release of ailerons decreases the degree of lateral stability of swept-back wing airplane and does not affect lateral

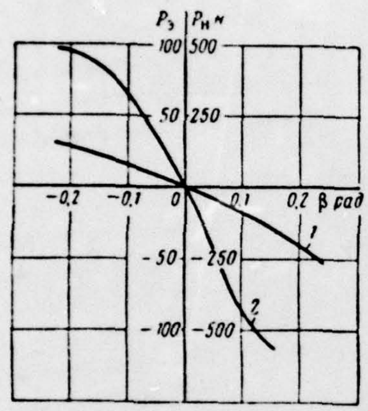
stability of straight-wing airplane. To the weather cock is the reliability of straight-wing airplane. To the weathercock stability of straight-wing airplane. On weathercock stability the release of ailercns in effect does not have an effect.

For a force on steering control it is possible to obtain simpler expression, if we into formulas (13.27) substitute the value of from relationships (13.5) and to consider relationship (13.33):

$$P_{э} = -k_{ш.э} S_{э} b_{э} q \frac{m_{ш}^{\delta_{э}}}{m_{x}^{\delta_{э}}} \left(m_{x\,св}^{\beta} - \frac{m_{x}^{\bar{\delta}_{н}}}{m_{y}^{\delta_{н}}} m_{y}^{\beta} \right) \beta. \qquad (13.34)$$

Fig. 13.14. Approximate balancing diagram of forces in flight with the slip: 1 - on steering control; 2 - on pedals.

Key: (1). rad.



It is obvious, from formulas (13.31), (13.34) are determined the forces, which must be applied to pedals and steering control for the retention of ailerons and rudder in the deflected position, caused by requirements for the balance of the lateral torque/moments in flight with slip. These values are called balancing, and the curves of the dependences of $P_H, P_э$ on angle are balancing diagrams (balancing curves) on forces to the levers of the lateral control (Fig. 13.14).

The balancing curves on forces are the important characteristics of the lateral static controllability. On the quality of controllability, the pilot judges in essence by those forces which for him he is necessary to apply to control levers for the balance of torque/moments or transition from one flight conditions toward another. Based on this, balancing curves must satisfy the certain conditions, outlined below.

1. The maximum forces, applied by pilot to levers, must correspond to his physical possibilities, for which in all range of the possible slip angles necessary the execution of the inequalities:

$$P_{э\,max} \leqslant 196\ н\,(20\ кГ);$$
$$P_{н\,max} \leqslant 1470 \div 1960\ н\,(150 \div 200\ кГ).$$

[н = N; кГ = kg]

2. So as in the case of handling in pitch, the lateral forces must impede a change in the parameter, which characterizes flight conditions. Consequently, to a positive increase in the slip angle must correspond the negative increase of pedal forces and the positive increase of forces on steering control, i.e., must be made conditions

$$\frac{dP_н}{d\beta} < 0; \quad \frac{dP_э}{d\beta} > 0.$$

. Page 263. According to relationships (13.31) and (13.34) these conditions are satisfied if

$$m^{\beta}_{y\,\text{н.св}}<0,\quad m^{\beta}_{x\,\text{э.св}}-\frac{m_x^{\delta_\text{н}}}{m_y^{\delta_\text{н}}}m^{\beta}_y<0,$$

i.e. if aircraft is weather cock stable with the freed rudder and if

$$|m^{\beta}_{x\,\text{э.св}}|>\frac{m_z^{\delta_\text{н}}}{m_y^{\delta_\text{н}}}|m^{\beta}_y|.$$

3. If the permissible transverse acceleration of the aircraft of n_z is restricted to the conditions of the comfort (for passenger aircraft) or of strength, then output/yield to such flight conditions must be hinder/hampered by the need for application/appendix to the control levers of large forces. As evaluation criteria of this property of aircraft serves the gradient of force on the transverse acceleration:

$$\left.\begin{aligned} P_{э}^{n_z} &= \frac{\partial P_{э}}{\partial n_z}, \\ P_{н}^{n_z} &= \frac{\partial P_{н}}{\partial n_z}. \end{aligned}\right\} \qquad (13.35)$$

Taking into consideration that $n_z = \frac{c_z q S}{G} \approx \frac{c_z}{c_y}$, for these indices from formula (13.31) and (13.34) can be obtained

$$\left.\begin{aligned} P_{н}^{n_z} &= k_{ш.н} k_{в.о} S_{н} b_{н} \frac{G}{S} \frac{m_{ш}^{\delta_{н}}}{m_{y}^{\delta_{н}}} \frac{m_{y\ н\cdot св}^{\beta}}{c_z^{\beta}}, \\ P_{э}^{n_z} &= k_{ш.э} S_{э} b_{э} \frac{G}{S} \frac{m_{ш}^{\delta_{э}}}{m_{x}^{\delta_{x}}} \frac{1}{c_z^{\beta}} \left(m_{x\ св}^{\beta} - \frac{m_{x}^{\delta_{н}}}{m_{y}^{\delta_{н}}} m_{y}^{\beta} \right). \end{aligned}\right\} \qquad (13.36)$$

The maximum permissible transverse acceleration from the conditions of the comfort of the passengers it must not exceed value 0.3-0.4. Consequently, forces on steering control and pedals at these

sizes of the g-force must reach the values, close to maximum. Relieve of forces from the levers of the lateral control in the assigned flight conditions is realized by aerodynamic trim tabs. The size/dimensions of the rudder trim tab and aerolons select on the basis of the conditions of the possibility of relieving forces from pedals and from steering control under the most adverse flight conditions (failure of one engine, flight in a slip with high cross wind, etc.).

On the supercritical Mach numbers, the forces considerably increase as a result of intense an increase in the $m_{ш}^{\delta_{н}}$ and $m_{ш}^{\delta_{э}}$.

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Obtaining the acceptable characteristics of the lateral static controllability with forces over a wide range of flight speeds during direct control of the organ/controls of the lateral control is impossible. Therefore in the system of lateral control, just as longitudinal, widely are applied booster drives.

PROBLEMS FOR REPETITION.

1. By bank to which wing does react stable aircraft with slip?

2. In what the difference between the weather cock and path (course) stability?

3. Which structural/design means do make it possible to ensure transverse and weathercock stability?

4. Why in the balanced rectilinear flight in a slip is necessary the deflection of both ailerons and rudder?

5. Why is possible the setting up of trim tab only on one of the ailercns?

6. In what the sense of the gradient of force on the transverse acceleration of of as criterion for controllability?

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